REPORT
FORCE CONSTANT OF A SPRING

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Experimental results

Different masses have been hung on a spring, and the relationship between the strength of the masses and the deformation of the spring has been studied, in order to verify the Hooke’s law and to calculate the force constant of the spring. Three measurements of length for each mass have been taken, computing the average and the error of each one, correctly rounding all the values. Mass (m) and \( \pi \) have been considered without error, and the gravity (g) has been taken as \( g=9,8\pm0,1 \text{ m/s}^2 \). Resolution of ruler has been taken as 0,1 cm. Measured values are reported on next table:

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>L1 (cm)</th>
<th>L2 (cm)</th>
<th>L3 (cm)</th>
<th>Average L (cm)</th>
<th>( \Delta_{\text{A}}L ) (cm)</th>
<th>( \Delta_{\text{B}}L ) (cm)</th>
<th>( \Delta L ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>29,4</td>
<td>29,3</td>
<td>29,4</td>
<td>29,37</td>
<td>0,033</td>
<td>0,03</td>
<td>0,033</td>
</tr>
<tr>
<td>200</td>
<td>33,6</td>
<td>33,5</td>
<td>33,4</td>
<td>33,50</td>
<td>0,058</td>
<td>0,03</td>
<td>0,058</td>
</tr>
<tr>
<td>300</td>
<td>38,0</td>
<td>38,0</td>
<td>38,0</td>
<td>38,00</td>
<td>0</td>
<td>0,03</td>
<td>0,03</td>
</tr>
<tr>
<td>400</td>
<td>42,9</td>
<td>43,0</td>
<td>43,0</td>
<td>42,97</td>
<td>0,033</td>
<td>0,03</td>
<td>0,033</td>
</tr>
<tr>
<td>500</td>
<td>47,0</td>
<td>46,7</td>
<td>46,7</td>
<td>46,80</td>
<td>0,10</td>
<td>0,03</td>
<td>0,10</td>
</tr>
<tr>
<td>600</td>
<td>51,0</td>
<td>51,2</td>
<td>51,2</td>
<td>51,13</td>
<td>0,067</td>
<td>0,03</td>
<td>0,067</td>
</tr>
</tbody>
</table>

\[
AvL = \frac{\sum_{i=1,2,3}L_i}{3}, \quad \Delta_{\text{A}}L = \frac{0,1}{2\sqrt{3}}=0,03, \quad \Delta_{\text{B}}L = \sqrt{\frac{\sum_{i=1,2,3}(L_i-AvL)^2}{3-2}}, \quad \Delta L = \max(\Delta_{\text{A}}L,\Delta_{\text{B}}L)
\]
Drawing \( L \) versus \( m \), performing a linear fitting and using the function “Estimación Lineal” to get the errors of linear fitting, we obtain (error bars are so small that they can’t be seen on the graph):

\[
\begin{array}{c|c|c}
\text{Estimacion Lineal. Results} & \text{Error} \\
\hline
\text{Slope (cm/g)} & 0,04379 & 0,00069 \\
\text{L_0 (cm)} & 24,95 & 0,27 \\
\text{R^2} & 0,999 \\
\end{array}
\]

\[
\text{Slope} = 0,04379\pm 0,00069 \text{ cm/g} = 0,4379\pm 0,0069 \text{ m/Kg}
\]

From slope:

\[
k = \frac{g}{\text{Slope}} = \frac{9,8}{0,4379} = 22,3795 \text{ N/m}
\]

and taking into account the error propagation:

\[
\Delta k = \sqrt{\left(\frac{\partial k}{\partial g}\right)^2 \Delta g^2 + \left(\frac{\partial k}{\partial \text{Slope}}\right)^2 \Delta \text{Slope}^2} = \sqrt{\frac{1}{\text{Slope}^2} \Delta \text{Slope}^2 + \frac{g^2}{\text{Slope}^2} \Delta \text{Slope}^2} = \sqrt{\frac{1}{0,4379^2} 0,1^2 + \frac{9,8^2}{0,4379^2} 0,0069^2} = 0,42 \text{ N/m}
\]

Therefore, the force constant of the spring is:

\[
k = 22,38 \pm 0,42 \text{ N/m}
\]

**Discussion and Conclusions**

The used masses lead to the linear range of the spring, because no deviation from linear behaviour has been observed.

The method used leads to good results because the relative error is low (\( \varepsilon = 0,42/22,38 = 0,02 = 2 \% \)), and correlation coefficient \( R^2 \) of linear fitting is close to 1. Anyway, the result could be strongly influenced not only by the accuracy of measurements, but by the values taken for several constants. For example, \( g \) has been taken with an error of 0,1 (\( g = 9,8 \pm 0,1 \)) and \( \pi \) without error, but if we had taken 9,81 m/s\(^2\) for \( g \) (\( \Delta g = 0,01 \)) then the error of \( k \) would be smaller.

Finally, from the obtained results, we can state that the method we have used is reasonably useful for determining the force constant of a spring.