UNIVERSIDAD POLITECNICA DE VALENCIA

Second modular exam FFI
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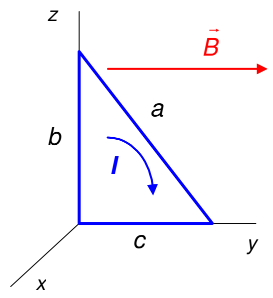
Applied Physics Dept.

1. Along the loop on picture, with sides ***a***, ***b*** and ***c*** (**b=4 m, c=3 m**) flows an intensity of current **I=2 A**. The loop is placed inside a uniform magnetic field $\vec{\mathbf{B}} = 5\vec{\mathbf{j}}\ \mathbf{T}$. Find: a) magnetic force acting on each side of loop. b) magnetic moment of loop. c) resulting torque on loop. **2 points**	**1.** Por la espira de la figura, de lados ***a, b*** y ***c*** (**b=4 m, c=3 m**), circula una corriente **I=2 A.** La espira se encuentra dentro de un campo magnético uniforme $\vec{\mathbf{B}} = 5\vec{\mathbf{j}}\ \mathbf{T}$. Calcula: a) la fuerza magnética que aparece sobre cada uno de los lados de la espira. b) el momento magnético de la espira. c) el momento resultante de las fuerzas sobre la espira. **2 puntos**	

a) The force on a conductor of length $\vec{\ell}$ with a current I inside a magnetic field $\vec{B}$ is $\vec{F} = I(\vec{\ell}\times\vec{B})$. Therefore on each side of loop we'll have:

$$\vec{F}_a = 2(\vec{a}\times\vec{B}) = 2((-4\vec{k}+3\vec{j})\times 5\vec{j}) = 40\vec{i}\ \text{N}$$
$$\vec{F}_b = 2(4\vec{k}\times 5\vec{j}) = -40\vec{i}\ \text{N}$$
$$\vec{F}_c = 2(-3\vec{j}\times 5\vec{j}) = 0$$

b) Magnetic moment of a loop is defined as $\vec{\mu} = I\vec{S}$. As $\vec{S} = \frac{4\cdot 3}{2}(-\vec{i}) = -6\vec{i}\ \text{m}^2$

$$\vec{\mu} = I\vec{S} = 2(-6\vec{i}) = -12\vec{i}\ \text{Am}^2$$

c) And the torque

$$\vec{\tau} = \vec{\mu}\times\vec{B} = -\frac{1}{2}Icb\vec{i}\times B\vec{j} = -\frac{1}{2}IcbB\vec{k} = -60\vec{k}\ \text{Nm}$$

2. State Ampère's law and apply it to compute the magnetic field created by an infinite straight carrying current wire, flowed by an intensity of current **I**, on a point placed at a distance **x** from wire. Explain the computations done. **2 points**	**2.** Enuncia el teorema de Ampère y aplícalo para calcular el campo magnético creado por un conductor rectilíneo e indefinido, recorrido por una intensidad de corriente **I**, en un punto situado a una distancia **x** del conductor. Explica los cálculos realizados. **2 puntos**

Ampére's law statement appears on point 7.5 of notes:
"The circulation of magnetic field vector along any enclosed curve equals the product of the constant μ_0 by the addition of the intensities of current crossing any surface bordered by the curve. The sign of the intensity will be positive when it was in accordance with the screw or the right hand rule with the sense of the circulation, and negative in another case."

$$C = \oint \vec{B}d\vec{l} = \mu_0 \sum I$$

Related to the magnetic field created by a straight carrying current wire, on a point placed at a distance x, appears on next page:

In order to apply Ampère's law, we choose a circumference of radius *x*, perpendicular to conductor and centered on a point of such conductor. This circumference is a field line of magnetic field, being the magnetic field vector tangent to this line at any point. On the other hand, as distance to conductor is the same for all the points of line, modulus of magnetic field will also be the same. So, circulation of magnetic field along this circumference is $C = \oint \vec{B}d\vec{l} = B2\pi x$

Considering the surface of a disk bordered by this circumference, the intensity crossing this disk is *I* (positive because its sense is in accordance with the sense of circulation chosen). Then, applying Ampère's law becomes

$$C = \oint \vec{B}d\vec{l} = B2\pi x = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi x}$$

3. A conductor rod with resistance **R** and length **L** is gliding with speed **v** along a conductor U shaped, as it's shown on picture. This set is perpendicullary placed into a uniform and stationary magnetic field **B**. Compute:

a) Magnetic flux through the loop as a function of **x**.

b) Induced electromotive force on loop.

c) Intensity of current flowing along the rod, giving and clearly explaining its sense.

2 points

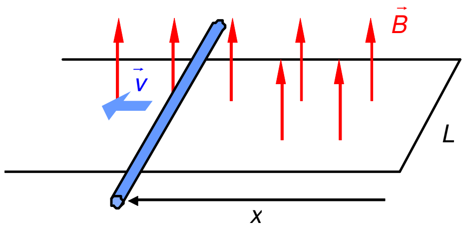
3. Una barra conductora de resistencia **R** y longitud **L** desliza con velocidad **v** sobre un conductor en forma de U, como se ve en la figura. El conjunto se encuentra dentro de un campo magnético **B** uniforme y estacionario, perpendicular a la espira. Calcula:

a) Flujo del campo magnético a través de la espira en función de **x**.

b) La fuerza electromotriz inducida en la espira.

c) La corriente que circula por la barra, indicando clara y razonadamente su sentido.

2 puntos



Solution:

a) $\phi = \vec{B} \cdot \vec{S} = BLx \; Wb$

b) Both B and L are not depending on time; then $|\varepsilon| = \left|\frac{d\phi}{dt}\right| = BL\frac{dx}{dt} = BLv \; V$

c) $i = \frac{\varepsilon}{R} = \frac{BLv}{R} \; A$

Magnetic flux through the loop is crossing the loop from down to up; as surface of loop is increasing on time, the magnetic flux will also increase on time. So, the induced current should create a magnetic flux from up to down; the induced current must flow along the loop in <u>clockwise sense</u>.

4. On a RLC series circuit, with a **R=2Ω** resistor, a **L=3 mH** inductor and a **C=400 μF** capacitor, drop of potential on terminals of inductor is $\mathbf{u_L(t) = 30\cos\left(500t + 30^\circ\right)}$ **V**.

Compute intensity of current, drop of potential on terminals on each device, and drop of potential on terminals of RLC series circuit.

2 points

4. En un circuito RLC serie, con una resistencia **R=2Ω**, una bobina **L=3 mH** y un condensador **C=400 μF**, la tensión en bornes de la autoinducción es $\mathbf{u_L(t) = 30\cos\left(500t + 30^\circ\right)}$ **V**.

Calcula la intensidad de corriente, la caída de tensión en cada elemento y la caída de tensión total en los terminales del circuito RLC.

2 puntos

As we know drop of potential on terminals of <u>inductor</u>, the amplitude of intensity of current flowing along the inductor will be $X_L = L\omega = 3 \cdot 10^{-3} \cdot 5 \cdot 10^2 = 1{,}5 \; \Omega$ $\quad I_m = \frac{U_{Lm}}{X_L} = \frac{30}{1{,}5} = 20 \; A$

Phase lag between drop of potential and intensity will be 90º: $90 = \varphi_u - \varphi_i = 30 - \varphi_i \Rightarrow \varphi_i = 30 - 90 = -60^\circ$

Then: $i(t) = 20\cos(500t - 60^\circ) \; A$

This intensity is the same for all the devices in series (R,L and C). Taking in account that phase lag between drop of voltatge and intensity on <u>capacitor</u> is -90º, and amplitude of voltatge is related to the capacitive reactance, it follows:

$X_C = \frac{1}{C\omega} = \frac{1}{400 \cdot 10^{-6} \cdot 5 \cdot 10^2} = 5 \; \Omega$ $\quad U_{Cm} = I_m X_C = 20 \cdot 5 = 100 \; V$

$-90 = \varphi_u - \varphi_i = \varphi_u - (-60) \Rightarrow \varphi_u = -90 - 60 = -150^\circ$ and $u_c(t) = 100\cos(500t - 150^\circ) \; V$

On <u>resistor</u>: $U_{Rm} = I_m R = 20 \cdot 2 = 40 \; V$ $\quad 0 = \varphi_u - \varphi_i = \varphi_u - (-60) \Rightarrow \varphi_u = -60^\circ$

$u_R(t) = 40\cos(500t - 60^\circ) \; V$

Concerning the drop of potential on <u>terminals of RLC circuit</u>:

$Z = \sqrt{2^2 + (1{,}5 - 5)^2} = 4 \; \Omega$ $\quad U_m = I_m Z = 20 \cdot 4 = 80 \; V$

$tg\varphi = \frac{1{,}5 - 5}{2} = -1{,}75 \Rightarrow \varphi = -60^\circ = \varphi_u - (-60) \Rightarrow \varphi_u = -60 - 60 = -120^\circ$

$u(t) = 80\cos(500t - 120^\circ) \; V$

5. A n extrinsic semiconductor is made of silicon doped with $4{\cdot}10^{20}$ atoms of Arsenic/cm^3 (donor). If intrinsic carrier density of silicon at 300 K is $n_i = 1{,}5{\cdot}10^{16}$ cm^{-3}: a) Which is the density of electrons (n) and holes (p) in this semiconductor at 300 K? b) Which would be the density of electrons and holes at 300 K if semiconductor wasn't doped? **2 points**	**5.** Un semiconductor extrínseco tipo n está formado por silicio con un dopado de $4{\cdot}10^{20}$ átomos de Arsénico/cm^3 (donador). Si la concentración intrínseca del silicio a 300 K es $n_i = 1{,}5{\cdot}10^{16}$ cm^{-3}: a) ¿Cuál es la concentración de electrones (n) y huecos (p) en este semiconductor a 300 K? b) ¿Cuál sería la concentración de electrones y huecos a 300 K si el semiconductor no estuviera dopado? **2 puntos**

a) As semiconductor is doped with a density of donor atoms 10^4 times the intrinsic carrier density, we can suppose that the density of electrons equals the donor density:

$n \approx N_D = 4 \cdot 10^{20}\ e/cm^3$ And $p = \frac{n_i^2}{n} = \frac{(1{,}5 \cdot 10^{16})^2}{4 \cdot 10^{20}} = 1{,}12 \cdot 10^{12}\ holes/cm^3$

If semiconductor wasn't doped, the density of electrons and holes at 300 K would be the intrinsic carrier density: $n = p = 1{,}5 \cdot 10^{16}\ carrier/cm^3$

FORM

Magnetic Forces $\vec{F} = q(\vec{v} \times \vec{B})$ $d\vec{F} = I d\vec{l} \times \vec{B}$ $\vec{\mu} = N \cdot I \cdot \vec{S}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$ $V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$

Sources of magnetic field $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$ $\mu_0 = 4\pi 10^{-7}$ (I.S.units) $B = \frac{\mu_0 I}{2\pi x}$

$B = \frac{\mu_0 I}{2R}$ $\oint_L \vec{B} \cdot \vec{dl} = \mu_0 \sum I$ $B = \frac{\mu_0 N I}{l}$

Electromagnetic induction $|\varepsilon| = \frac{d\phi}{dt}$ $\phi = L \cdot I$ $\phi_{21} = M \cdot I_1$ $W_L = \frac{1}{2} L \cdot I^2$

Alternating current $\varphi = \varphi_u - \varphi_i$ $X_L = L\omega$ $X_C = \frac{1}{C\omega}$ $U_{rms} = \frac{U_m}{\sqrt{2}}$ $I_{rms} = \frac{I_m}{\sqrt{2}}$

$\mathrm{tg}\varphi = \frac{L\omega - 1/C\omega}{R}$ $Z = \frac{U_m}{I_m} = \sqrt{R^2 + (L\omega - 1/c\omega)^2}$

$P(t) = u(t) \cdot i(t) = U_m I_m \cos\varphi \sin^2 \omega t + \frac{U_m I_m}{2} \sin\varphi \sin\omega t$ $P = U_{rms} I_{rms} \cos\varphi$

Semiconductors $n \cdot p = n_i^2$ $N_A + n = N_D + p$