First mid term FFI exam
December, 1th , 2014
Year 2014/15

Applied Physics Dept.

1. (2,5 points) Given the point charges -4Q and Q on picture, placed at points A(0,0) m and B(4,0) m:

a) (1) Find a point P of X axis where the electric fields due to both charges are cancelled (total electric field zero).

b) (1) Find a point P' of X axis where the electric potentials due to both charges are cancelled (total electric potential zero).

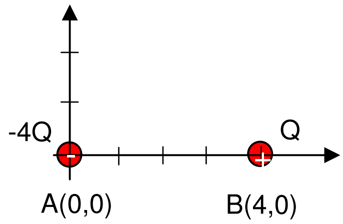
c) (0,5) Compute the work done by the forces of the electric field to carry a -Q point charge from above point P to point P'.

1. (2,5 puntos) Dadas las cargas puntuales -4Q y Q de la figura, situadas en los puntos A(0,0) m y B(4,0) m:

a) (1) Encuentra un punto P del eje X donde el campo eléctrico total sea cero.

b) (1) Encuentra un punto P' del eje X donde el potencial eléctrico total sea cero.

c) (0,5) Calcula el trabajo hecho por las fuerzas del campo debido a ambas cargas para llevar una carga –Q desde el punto P hasta el punto P'.



a) Electric field can only be zero at points placed on right of point B. On left to A, electric field due to -4Q is always stronger thatn electric field due to Q, and between A and B, both electric fields are reinforced, never cancelled. If the coordinates of such point are (x,0), being x>4, then must be verified that

$$k\frac{4Q}{x^2}=k\frac{Q}{(x-4)^2}\Rightarrow x=8\ m \quad \text{Therefore} \quad P(8,0)\ m$$

b) The electric potential can only be zero at points on right of B or between A and B (electric potential is an scalar). If we consider the point P' on right of B, with coordinates (x,0), being x>4, must be verified that

$$k\frac{4Q}{x}=k\frac{Q}{(x-4)}\Rightarrow x=\frac{16}{3}\ m \quad \text{Therefore } P'(\frac{16}{3},0)\ m$$

There is a second correct solution by taking P' between A and B. If (x,0) are the coordinates of P'

$$k\frac{4Q}{x}=k\frac{Q}{(4-x)}\Rightarrow x=\frac{16}{5}\ m \quad \text{Therefore } P'(\frac{16}{5},0)\ m$$

Anyone of both solutions is correct.

c) Potential on point P(8,0) is $V_P=k\frac{-4Q}{8}+k\frac{Q}{4}=-k\frac{Q}{4}V$ Then

$$W_{PP'}=-Q(V_P-V_{P'})=-Q(-k\frac{Q}{4}-0)=k\frac{Q^2}{4}\quad J$$

2. (1 point) A positive point charge Q is placed at the center of a hollow and electrically neutral and isolated sphere (radii R and 2R).

a) (0,3) Say which are the charges appearing on inner and outer surface of sphere.

b) (0,4) By applying Gauss's law, compute the electric field at a point P placed at a distance r from the center of sphere (r>2R).

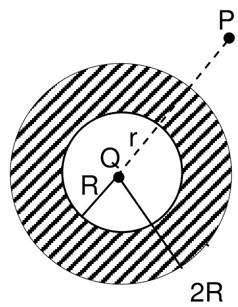
c) (0,3) Compute the electric potential of the sphere.

2. (1 punto) Una carga puntual Q positiva está colocada en el centro de una esfera hueca y eléctricamente neutra y aislada (radios R y 2R).

a) (0,3) Indica las cargas que aparecen en la superficie interior y en la superficie exterior de la esfera.

b) (0,4) Aplicando el teorema de Gauss, calcula el campo eléctrico en un punto P situado a una distancia r del centro de la esfera (r>2R).

c) (0,3) Calcula el potencial eléctrico de la esfera.



a) As there is total influence between charge and sphere, a charge –Q appears on inner surface of sphere. As the sphere is isolated and it's neutral a positive charge Q must appear on outer surface.

b) Due to the simmetry of problem, electric field at P will have the direction of radius and its modulus will only depend on the distance to the center of sphere. If we consider the surface of a sphere passing through P (radius r), the electric flux through such sphere is:

$$\phi = \int_{sphere} \vec{E}\cdot d\vec{S} = \int_{sphere} E\cdot dS = E\int_{sphere} dS = E4\pi r^2$$

According Gauss's law

$$\phi = E4\pi r^2 = \frac{Q+Q-Q}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

c) If we consider a point placed at the outer surface of sphere and we compute its difference of potential related to the infinite is: $V_{2R} - V_{\infty} = \int_{2R}^{\infty} \vec{E}d\vec{r} = \int_{2R}^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2}dr = -\frac{Q}{4\pi\varepsilon_0 r}\Big|_{2R}^{\infty} = \frac{Q}{8\pi\varepsilon_0 R}$

As $V_\infty=0$, $V_{sphere} = V_{2R} = \dfrac{Q}{8\pi\varepsilon_0 R}$

3. (2 points) Two parallel plate capacitors, initially discharged and with equal capacitance *C* are connected in series to a battery giving 5 V between its terminals.

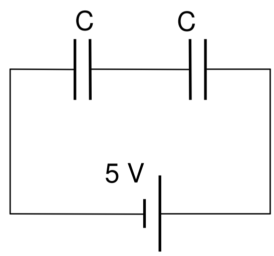
a) (0,8) Find the electric charge on each capacitor, Q_1 and Q_2.

b) (1,2) A dielectric with dielectric permitivitty $\varepsilon_r = 3$ is inserted between the plates of one of capacitors. Compute the new charges Q'_1 and Q'_2 and the new potential difference V'_1 and V'_2 on each capacitor.

3. (2 puntos) Dos condensadores planos, inicialmente descargados y de la misma capacidad *C*, están conectados en serie a una batería que da 5 V entre sus terminales

a) (0,8) Halla la carga en cada condensador, Q_1 y Q_2.

b) (1,2) Entre las placas de uno de los condensadores se introduce un dieléctrico de permitividad relativa $\varepsilon_r = 3$. Calcula las nuevas cargas Q'_1 y Q'_2 y las nuevas diferencias de potencial V'_1 y V'_2 en cada uno de los condensadores.



a) As both capacitors are connected in series, their charges are equal $Q_1=Q_2$. Then:

$$\frac{Q_1}{C} + \frac{Q_2}{C} = 5 \Rightarrow Q_1 = Q_2 = \frac{5C}{2}$$

b) When a dielectric is inserted, the capacitance of this capacitor is multiplied by ε_r. Let's suppose that dielectric is inserted on capacitor 1: $C'_1=3C$. As both capacitors have the same charge $Q'_1=Q'_2$:

$$\frac{Q'_1}{3C} + \frac{Q'_2}{C} = 5 \Rightarrow Q'_1 = Q'_2 = \frac{15}{4}C$$

$$V_1' = \frac{Q_1'}{3C} = \frac{5}{4}V \qquad V_2' = \frac{Q_2'}{C} = \frac{15}{4}V$$

4. (2 points) Four resistors are connected as can be seen on picture. Along R_4 flows an intensity of current I=2 A. Compute:

a) (0,2) The potential difference between points A and B, V_A-V_B.

b) (0,2) The intensity I_3 flowing along R_3.

c) (0,4) The potential difference between points A and C, V_A-V_C.

d) (0,6) The consumed power by the set of all resistors.

e) (0,6) If the efficiency of generator is 80%, find electromotive force and internal resistance of generator (ε, r).

4. (2 puntos) Cuatro resistencias están conectadas tal y como se ve en la figura. Por R_4 circula una corriente I=2 A. Calcular:

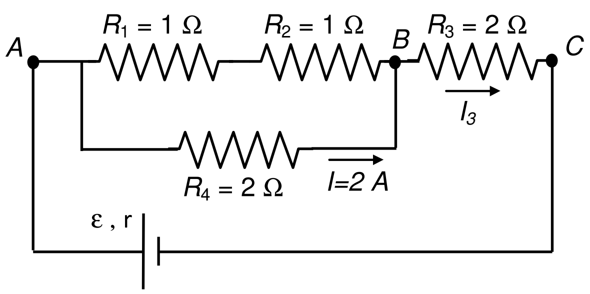
a) (0,2) La diferencia de potencial entre los puntos A y B, V_A-V_B.

b) (0,2) La intensidad I_3 que circula por R_3.

c) (0,4) La diferencia de potencial entre los puntos A y C, V_A-V_C.

d) (0,6) La potencia total consumida por el conjunto de todas las resistencias.

e) (0,6) Si el rendimiento del generador es del 80%, calcula su fuerza electromotriz y su resistencia interna (ε, r).



a) Potential difference on terminals of R_4 is $V_{R4} = V_A - V_B = I*R_4 = 2*2 = 4$ V

b) The intensity flowing along R_1 and R_2 is: $I_{12}=4/2=2$ A. As $I_3=I_{12}+I_4$ therefore $I_3=2+2=4$ A.

c) Potential difference between terminals of R_3 is V_{R3}=2*4=8 V. Therefore V_A-V_C=V_{R4}+V_{R3}=4+8=12 V

d) Equivalent resistance of the set of resistors is R_{eq}=3 Ω. The consumed power: $P_R = I_3^{\,2} R_{eq} = 4^2 \cdot 3 = 48\ w$

e) The supplied power to the circuit by the generator equals the consumed power by the set of resistors and then P_s=48 w. From efficiency

$$\eta_g = \frac{P_s}{P_g} \Rightarrow 0{,}8 = \frac{48}{P_g} \Rightarrow P_g = \frac{48}{0{,}8} = 60\ w \qquad \text{As } P_g = \varepsilon I \Rightarrow 60 = \varepsilon * 4 \Rightarrow \varepsilon = \frac{60}{4} = 15\ V$$

Lost power on internal resistance of generator is the difference between generated and supplied power $P_r = P_g - P_s = 60 - 48 = 12\ w$ And $P_r = I^2 r \Rightarrow 12 = 4^2 r \Rightarrow r = \frac{12}{16} = \frac{3}{4} = 0{,}75\ \Omega$

5. (2,5 points) Given the circuit on picture, compute:

a) (1,5) Intensity of current flowing along each branch of circuit with the shown senses, I_1, I_2 and I_3.

b) (1) Difference of potential between points B and D, V_B-V_D and Thevenin's equivalent generator between points B and D, clearly showing its polarity.

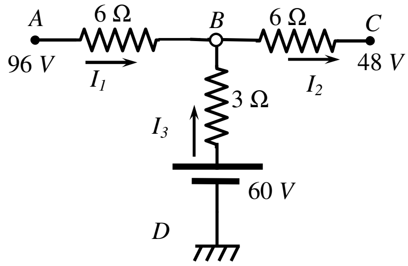
2,5 points

5. (2,5 puntos) Dado el circuito de la figura, calcula:

a) (1,5) La intensidad de corriente en cada rama con los sentidos mostrados, I_1, I_2 y I_3.

b) (1) La diferencia de potencial entre los puntos B y D, V_B-V_D,y el generador equivalente de Thevenin entre los puntos B y D, indicando claramente su polaridad.

2,5 puntos

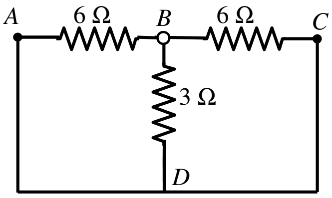


a) This is a network with 2 junctions and two loops, and so we'll need one equation for junctions and two equations for loops:

$$\left.\begin{array}{r} I_1 - I_2 + I_3 = 0 \\ V_{AD} = 96 = I_1 6 - I_3 3 - (-60) \\ V_{CD} = 48 = -I_2 6 - I_3 3 - (-60) \end{array}\right\} \Rightarrow I_1 = 5\ A \quad I_2 = 3\ A \quad I_3 = -2\ A$$

b) $V_B - V_D = -3I_3 - (-60) = -3(-2) + 60 = 66\ V$

Passive circuit and equivalent resistance between B and D (removing all the generators) are:



$$\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{3} \Rightarrow R_{eq} = 1{,}5\ \Omega$$

So, Thevenin's equivalent generator between B and D will be: