



1. (2 points) Two point charges $2 \mu\text{C}$ y $-3 \mu\text{C}$ sized are placed on vacuum respectively at points $\mathbf{A}(0,0) \text{ m}$ and $\mathbf{B}(3,0) \text{ m}$. Compute:

a) Resulting electric field at point $\mathbf{C}(0,4) \text{ m}$. Draw electric field vectors due to each charge and the resulting electric field.

b) The work done by the electric field when moving a $1 \mu\text{C}$ point charge from infinite to point C. Say you if this work is done by the forces of electric field or by external forces.

1. (2 puntos) Dos cargas puntuales de $2 \mu\text{C}$ y $-3 \mu\text{C}$ se encuentran en el vacío en las posiciones $\mathbf{A}(0,0) \text{ m}$ y $\mathbf{B}(3,0) \text{ m}$ respectivamente. Calcula:

a) El campo eléctrico resultante en el punto $\mathbf{C}(0,4) \text{ m}$. Dibuja los vectores campo eléctrico que crea cada carga y el campo eléctrico resultante.

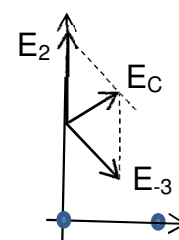
b) El trabajo realizado por el campo eléctrico al desplazar una carga de $1 \mu\text{C}$ desde el infinito al punto C. Decir si este trabajo es realizado por las fuerzas del campo eléctrico o por fuerzas externas a él.

$$\text{a) } \vec{E}_C = \vec{E}_2 + \vec{E}_{-3} = k \frac{2 \cdot 10^{-6}}{16} \vec{j} + k \frac{3 \cdot 10^{-6}}{25} \frac{(3\vec{i} - 4\vec{j})}{5} = 9 \cdot 10^3 \left(\frac{2}{16} \vec{j} + \frac{3}{25} \frac{3\vec{i} - 4\vec{j}}{5} \right) = 648\vec{i} + 261\vec{j} \text{ N/C}$$

$$\text{b) } V_C = V_{C_2} + V_{C_{-3}} = k \frac{2 \cdot 10^{-6}}{4} + k \frac{-3 \cdot 10^{-6}}{5} = 9 \cdot 10^3 \left(\frac{2}{4} - \frac{3}{5} \right) = -900 \text{ V}$$

$$W_{\infty C} = q(V_{\infty} - V_C) = 1 \cdot 10^{-6} (0 + 900) = 0,9 \cdot 10^{-3} \text{ J}$$

As the work is positive, it means that the work is done by the forces of electric field.



2. (1,5 points) State Gauss's law and apply it for computing the electric field produced by a sphere having radius R , charged with a **surface density of charge σ** , at a distance $2R$ from its center.

2. (1,5 puntos) Enuncia el teorema de Gauss y aplícalo para calcular el campo eléctrico creado por una esfera de radio R , cargada con una **densidad superficial de carga σ** , a una distancia $2R$ de su centro.

"The flux of the electric field through a closed surface S is equal to the net charge enclosed inside S

divided into ϵ_0 :
$$\int_S \vec{E} \cdot d\vec{S} = \frac{\sum Q}{\epsilon_0}$$
"

If we choose a gaussian sphere with radius $2R$ (its center being the same than the center of charged sphere), the surface vector at any point of this gaussian sphere will be parallel to the electric field vector. Besides, the modulus of this electric field (E) will be constant along the surface of the gaussian sphere. So, the flux of electric field through the gaussian sphere is:

$$\int_S \vec{E} \cdot d\vec{S} = \int_S E \cdot dS = E \int_S dS = ES = E4\pi(2R)^2 = 16\pi R^2 E$$

The net charge enclosed inside the gaussian sphere is the charge disatributed across the surface of charged

sphere: $Q = \sigma S = \sigma 4\pi R^2$ And applying Gauss's law: $16\pi R^2 E = \frac{\sigma 4\pi R^2}{\epsilon_0} \Rightarrow E = \frac{\sigma}{4\epsilon_0}$

3. (2 points) Picture shows 3 equal capacitors with capacitance C , connected to a potential difference V_0 .

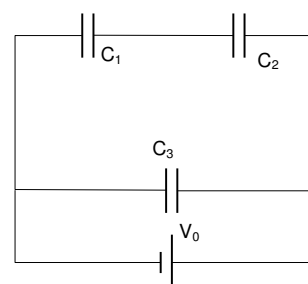
a) Find the electric charge on each capacitor.

b) **We remove the battery** and we insert a dielectric having a relative dielectric permittivity $\epsilon_r=4$ between the plates of capacitor C_3 . Find the electric charge on each capacitor after the dielectric has been inserted.

3. (2 puntos) La figura muestra 3 condensadores iguales de capacidad C , conectados a una diferencia de potencial V_0 .

a) Halla la carga en cada condensador.

b) **Retiramos la fuente** de tensión e introducimos un dieléctrico de permitividad relativa $\epsilon_r=4$ en el condensador C_3 . Halla la carga en cada condensador después de introducir el dieléctrico.



$$C_1 = C_2 = C_3 = C$$

a) Voltage on capacitor C_3 is V_0 . Therefore: $Q_3 = CV_0$

C_1 and C_2 are equal and so V_0 is equally distributed between them, being $V_0/2$ the voltage on terminals of each capacitor: $Q_1 = Q_2 = C \frac{V_0}{2}$

b) When a dielectric is inserted inside C_3 , the capacitance of this capacitor is multiplied by ϵ_r . Then $C'_3 = 4C$. When battery is removed, the total charge on capacitors C_1 and C_3 (or C_2 and C_3) is redistributed but it's preserved:

$$Q'_1 + Q'_3 = Q_1 + Q_3 = \frac{3}{2} CV_0 \quad (1)$$

Charge on capacitors C_1 and C_2 is the same, because they are connected in series: $Q'_1 = Q'_2$ (2)

and voltage on both branches is also the same:

$$\frac{Q'_3}{4C} = \frac{Q'_1}{C} + \frac{Q'_2}{C} \quad (3)$$

Solving this system of equations, from (2) and (3) it comes $\frac{Q'_3}{4C} = \frac{Q'_1}{C} + \frac{Q'_1}{C} = \frac{2Q'_1}{C} \Rightarrow Q'_3 = 8Q'_1$

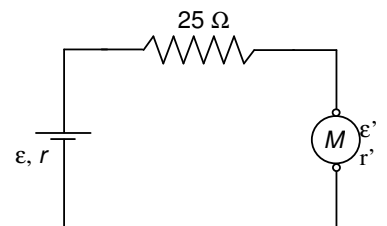
And using (1): $Q'_1 + Q'_3 = Q'_1 + 8Q'_1 = 9Q'_1 = \frac{3}{2} CV_0 \Rightarrow Q'_1 = \frac{1}{6} CV_0 = Q'_2$ $Q'_3 = 8Q'_1 = \frac{4}{3} CV_0$

4. (2 points) The engine of circuit consumes **110 W**, a 20% of them by Joule heating. If the battery supplies **210 W** to the circuit, determine:

- Consumed power on 25 Ω resistor.
- If the battery generates a power equal to **220 W**, determine the features of battery, ϵ and r .
- The features of engine, ϵ' and r' .

4. (2 puntos) El motor del circuito de la figura consume **110 W**, de los que un **20%** lo es por efecto Joule. Si la fuente suministra **210 W** al circuito, determina:

- Potencia consumida en la resistencia de 25 Ω .
- Si la fuente genera una potencia de **220 W**, determina las características de la fuente, ϵ y r .
- Las características del motor ϵ' y r' .



a) The power consumed on 25 Ω is the difference between the power supplied by the battery and the power consumed by the engine: $P_{25} = 210 - 110 = 100 \text{ W}$

b) From before computation, the intensity flowing along the circuit can be got:
 $P_{25} = 100 = I^2 25 \Rightarrow I = \sqrt{\frac{100}{25}} = 2 \text{ A}$

On battery $P_g = 220 = \epsilon I$ As $I = 2 \text{ A}$ $\epsilon = \frac{220}{2} = 110 \text{ V}$ On the other hand $P_s = 210 \text{ W}$. Then, the power consumed on internal resistance of battery (r) will be $P_r = P_g - P_s = 220 - 210 = 10 = I^2 r$

As $I = 2 \text{ A}$, $r = \frac{10}{4} = 2,5 \Omega$

c) On engine, the power consumed on its internal resistance (r') is a 20% of consumed power
 $P_{r'} = \frac{20 \cdot 110}{100} = 22 \text{ W}$

But $P_{r'} = I^2 r' \Rightarrow 22 = 2^2 r' \Rightarrow r' = \frac{22}{4} = 5,5 \Omega$

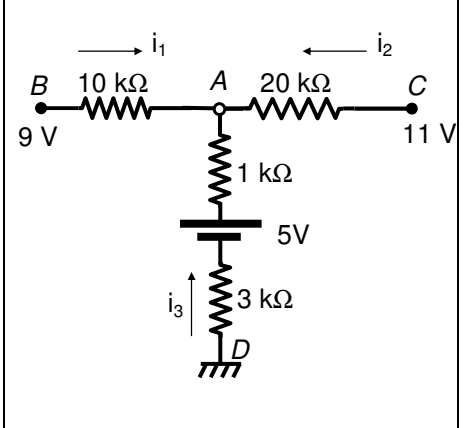
And the turned power will be a 80% of consumed power: $P_t = \frac{80 \cdot 110}{100} = 88 = \epsilon' I \Rightarrow \epsilon' = \frac{88}{2} = \frac{88}{2} = 44 \text{ V}$

5. (2,5 points) On circuit on picture, compute:

- Intensity of current flowing along each branch, with the shown directions.
- The equivalent Thevenin's generator between points A and B, clearly giving its polarity.
- The intensity of current that would flow along a $5\text{ k}\Omega$ resistor connected between A and B.

5. (2,5 puntos) Dado el circuito de la figura, calcula:

- La intensidad que circula por cada rama, con los sentidos indicados.
- El generador equivalente de Thevenin entre A y B, indicando claramente su polaridad.
- La intensidad de corriente que circularía por una resistencia de $5\text{ k}\Omega$ que conectásemos a los puntos A y B.

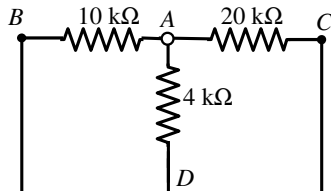


a) This is a network with 2 junctions and two loops, and so we'll need one equation for junctions and two equations for loops:

$$\left. \begin{aligned} I_1 + I_2 + I_3 &= 0 \\ V_{BD} = 9 &= I_1 \cdot 10 - I_3 \cdot 4 - (-5) \\ V_{CD} = 11 &= I_2 \cdot 20 - I_3 \cdot 4 - (-5) \end{aligned} \right\} \Rightarrow I_1 = \frac{9}{40} = 0,225\text{ mA} \quad I_2 = \frac{17}{80} = 0,2125\text{ mA} \quad I_3 = -\frac{7}{16} = -0,4375\text{ mA}$$

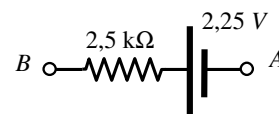
b) $V_A - V_B = -10I_1 = -10 \cdot \frac{9}{40} = -\frac{9}{4} = -2,25\text{ V}$

Passive circuit and equivalent resistance between A and B (removing all the generators) are:



$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{4} + \frac{1}{20} \Rightarrow R_{eq} = \frac{5}{2} = 2,5\text{ k}\Omega$$

So, Thevenin's equivalent generator between A and B will be:



c) If we connect a new $5\text{ k}\Omega$ resistor between A and B, by using Thevenin's equivalent generator, the intensity is:

$$I = \frac{2,25}{2,5 + 5} = \frac{2,25}{7,5} = 0,3\text{ mA}$$

