First mid term FFI exam
October, 26th , 2015
Year 2015/16

Applied Physics Dept.

1. (2,5 points) Given the point charges **3 μC** and **-3 μC** on picture, placed at points **(0,0) m** and **(0,4) m**:

a) (0,5) Compute the electric field vector due to both charges at point A, $\vec{E}_A$.

b) (0,5) Compute the electric potential due to both charges at point B, V_B.

c) (0,5) If it exists, find a point lying on Y axis where the total electric potential due to both charges was zero.

d) (0,5) Compute the work done by the forces of the electric field to carry a **2 μC** point charge from point B to point A.

e) (0,5) If it exists, find a point lying on Y axis where the electric field due to both charges was zero.

1. (2,5 puntos) Dadas las cargas puntuales de la figura, de **3 μC** y **-3 μC**, situadas en los puntos **(0,0) m** y **(0,4) m**:

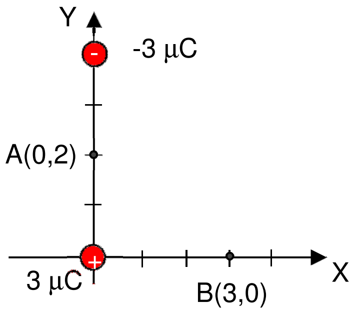
a) (0,5) Calcula el vector campo eléctrico debido a ambas cargas en el punto A, $\vec{E}_A$.

b) (0,5) Calcula el potencial electrostático debido a ambas cargas en el punto B, V_B.

c) (0,5) Si existe, encuentra un punto sobre el eje Y donde el potencial electrostático total debido a ambas cargas sea cero.

d) (0,5) Calcula el trabajo hecho por las fuerzas del campo para llevar una carga de **2 μC** desde el punto B hasta el punto A.

e) (0,5) Si existe, encuentra un punto sobre el eje Y donde el campo eléctrico total debido a ambas cargas sea cero.



To calculate the electric field or the electric potential, we'll apply the principle of superposition:

a) $\vec{E}_A = k(\frac{3\cdot 10^{-6}}{2^2}+\frac{3\cdot 10^{-6}}{2^2})\vec{j} = 9\cdot 10^9\frac{6\cdot 10^{-6}}{4}\vec{j} = \frac{27}{2}10^3\vec{j} = 13500\vec{j}\ N/C$

b) $V_B = k(\frac{3\cdot 10^{-6}}{3}-\frac{3\cdot 10^{-6}}{5}) = 9\cdot 10^9\cdot 3\cdot 10^{-6}(\frac{1}{3}-\frac{1}{5}) = 27\cdot 10^3\frac{2}{15} = \frac{54}{15}10^3 = 3600\ V$

c) That point is point A because $V_A = k(\frac{3\cdot 10^{-6}}{2}-\frac{3\cdot 10^{-6}}{2}) = 0$

d) $W_{BA} = q(V_B - V_A) = 2\cdot 10^{-6}(3600-0) = 7{,}2\cdot 10^{-3}\ J$

e) There isn't any point where electric field was zero. Only at infinite electric field is null.

2. (2 points) The picture shows a hollow and conductor sphere (inner radius R_1 and outer radius R_2) linked to ground. There is a positive point charge **Q** at the centre of sphere.

a) (0,5) ¿Which is the induced charge on inner and outer surfaces of sphere?

b) (0,5) Compute the electric field and the electrostatic potential at a point outer to the sphere, placed at a distance **r** from the centre ($r \geq R_2$).

c) (0,5) Compute the electric field and the electrostatic potential on the conductor sphere ($R_1 \leq r \leq R_2$).

d) (0,5) Compute the electric field and the electrostatic potential at a point inner to the sphere, placed at a distance **r** from the centre ($r \leq R_1$).

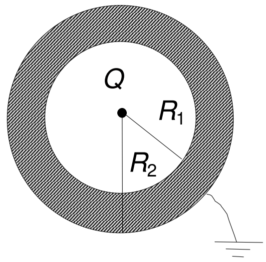
2. (2 puntos) La figura muestra una esfera hueca conductora de radio interior R_1 y exterior R_2, y conectada a tierra. Se coloca una carga puntual positiva, ***Q***, en el centro de la esfera.

a) (0,5) ¿Cuál es la carga inducida en las superficies interior y exterior de la esfera?

b) (0,5) Calcula el campo eléctrico y el potencial electrostático en un punto exterior a la esfera situado a una distancia **r** del centro ($r \geq R_2$).

c) (0,5) Calcula el campo eléctrico y el potencial electrostático en la esfera conductora ($R_1 \leq r \leq R_2$).

d) (0,5) Calcula el campo eléctrico y el potencial electrostático en un punto interior a la esfera situado a una distancia **r** del centro ($r \leq R_1$).



a) There is total influence between the charge and the inner surface of sphere. So, on inner surface of sphere is induced a charge –Q. As the sphere is connected to ground, on outer surface of sphere isn't induced any charge.

b) The electric potential and the electric field are zero at any point having $r>R_2$, because there isn't any charge outside of the sphere and the sphere acts as a Faraday's cage.

c) The electric field on the sphere is zero because it is a conductor in electrostatic equilibrium. The electric potential is zero because the sphere is linked to ground.

d) At a point inside the hollow of the sphere ($r<R_1$), the calculation of electric field can be done by applying Gaus's law. If we take a virtual sphere with radius r, the electric flux through this virtual sphere is

$$\phi = \int_{sphere} \vec{E}\cdot d\vec{S} = \int EdS = E\cdot S = E\cdot 4\pi r^2$$

And according Gaus's law: $\phi = E\cdot 4\pi r^2 = \dfrac{Q}{\varepsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

The electric potential at a point inside the hollow ($r\leq R_1$) can be computed through the difference of potential between a point with radius r and the sphere (radius R_1) and taking in account that the potential of sphere is zero:

$$V_r - V_{R_1} = V_r = \int_{sphere} \vec{E}\cdot d\vec{r} = \int Edr = \int_r^{R_1} \frac{Q}{4\pi\varepsilon_0 r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)_r^{R_1} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{R_1}\right) = \frac{Q}{4\pi\varepsilon_0 r} - \frac{Q}{4\pi\varepsilon_0 R_1}$$

3. (2,5 points) Two capacitors (capacitances **C** (1) and **2C** (2)) are connected to a power supply giving a difference of potential $\mathbf{V_0}$ between its terminals.

a) (0,6) Compute the charge taken by each capacitor, Q_1 and Q_2.

The power supply is removed and later a dielectric having a relative dielectric permittivity $\boldsymbol{\varepsilon_r=2}$ is inserted between the plates of capacitor (2). Compute:

b) (0,3) The new capacitance of capacitor (2), C'.

c) (0,6) The charge taken by each capacitor, Q'_1 and Q'_2.

d) (0,6) The difference of potential between the plates of both capacitors, V'.

e) (0,4) The energy stored on capacitor (2).

3. (2,5 puntos) Dos condensadores (capacidades **C** (1) y **2C** (2)) se conectan a una fuente de tensión que da una diferencia de potencial $\mathbf{V_0}$ entre sus terminales

a) (0,6) Calcula la carga en cada condensador, Q_1 y Q_2.

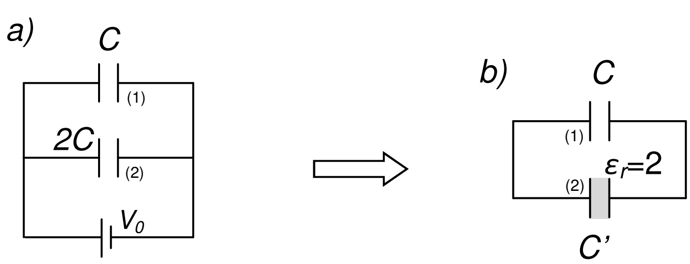
Se retira la fuente y después se inserta un dieléctrico de permitividad dielèctrica relativa $\boldsymbol{\varepsilon_r=2}$ entre las placas del condensador (2). Calcula:

b) (0,3) La nueva capacidad del condensador (2), C'.

c) (0,6) La carga en cada condensador, Q'_1 y Q'_2.

d) (0,6) La diferencia de potencial entre las placas de ambos condensadores, V'.

e) (0,4) La energía almacenada en el condensador (2).



a) Both capacitors are connected to a battery giving a difference of potential V_0 between its terminals.

So: $Q_1 = CV_0$ and $Q_2 = 2CV_0$

b) $C' = \varepsilon_r 2C = 2\cdot 2C = 4C$

c) When the power supply is disconnected and the dielectric is inserted, the charge on both capacitors is redistributed according the new capacitances. Moreover, the difference of potential will be the same for both capacitors, in such way that:

$$Q_1 + Q_2 = Q'_1 + Q'_2 \quad \text{and} \quad V' = \frac{Q'_1}{C} = \frac{Q'_2}{C'}$$

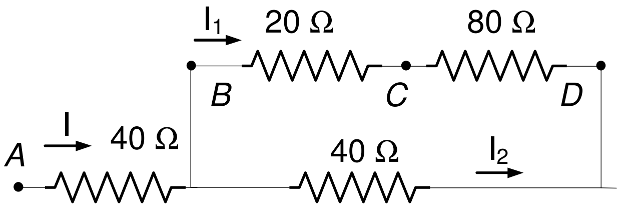
By solving this system with the results got on a) and b), it comes: $Q'_1 = \frac{3}{5}CV_0$ and $Q'_2 = \frac{12}{5}CV_0$

d) The difference of potential between the plates is the same for both capacitors:

$$V' = \frac{Q'_1}{C} = \frac{Q'_2}{4C} = \frac{3}{5}V_0$$

e) The energy stored on capacitor (2) is: $W'_2 = \frac{1}{2}C'V'^2 = \frac{1}{2}4C\frac{3^2}{5^2}V_0^2 = \frac{18}{25}CV_0^2$

4. (2 points) Given the association of resistors on figure, it's known that V_{BC} = **4 V**. Compute I_1, I_2, I, V_{CD}, V_{BD}, V_{AB} (0,3 each) and V_{AD} (0,2).	**4.** (2 puntos) En la asociación de resistencias de la figura, se sabe que V_{BC} = **4 V**. Calcula I_1, I_2, I, V_{CD}, V_{BD}, V_{AB} (0,3 cada resultado) y V_{AD} (0,2).



By applying Ohm's law to 20 Ω resistor: $V_{BC} = 4 = I_1 20 \Rightarrow I_1 = \frac{4}{20} = \frac{1}{5}\,A$

As I_1 is also the intensity flowing along 80 Ω resistor: $V_{CD} = I_1 80 = 16\,A$

and therefore $V_{BD} = V_{BC} + V_{CD} = 4 + 16 = 20\,A$ From this result I_2 can be got : $I_2 = \frac{V_{BD}}{40} = \frac{20}{40} = \frac{1}{2}\,A$

$I = I_1 + I_2 = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}\,A$ and $V_{AB} = I \cdot 40 = \frac{7}{10}40 = 28\,V$ $V_{AD} = V_{AB} + V_{BD} = 28 + 20 = 48\,V$

5. (1 point) Effective communication. Shortly give the basis, theorem or principle used for solving an exercise, and to do a tidy exam will weight up to1 point.	**5.** (1 punto) Comunicación efectiva. Indicar de manera escueta las bases, teorema o principio utilizado para resolver un ejercicio, y hacerlo de manera clara y organizada, contará hasta 1 punto.

Form

Electrostatics

$\vec{F} = K\frac{q_1 q_2}{r^2}\vec{u}_r$ $\quad \vec{E} = \frac{\vec{F}}{q}$ $\quad K = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 (S.I.)$ $\quad V_A - V_B = \int_A^B \vec{E}\cdot\vec{dr}$

$\vec{E} = K\frac{q}{r^2}\vec{u}_r$ $\quad V = K\frac{q}{r}$ $\quad \int_S \vec{E}\cdot\vec{dS} = \frac{\sum Q}{\varepsilon_0}$ $\quad W_{AB} = q(V_A - V_B)$

Conductors and capacitors

$E = \frac{\sigma}{\varepsilon_0}$ $\quad C = \frac{Q}{V}$ $\quad C = \frac{\varepsilon_0 S}{d}$

$C_{eq} = \sum C_i$ $\quad \frac{1}{C_{eq}} = \sum\frac{1}{C_i}$ $\quad E_d = \frac{E}{\varepsilon_r}$ $\quad C_d = \varepsilon_r C$ $\quad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$

Direct Current

$\vec{J} = n \cdot e \cdot \vec{v}_a$ $\quad \vec{J} = \sigma \cdot \vec{E}$ $\quad R = \frac{V_1 - V_2}{I}$ $\quad R = \rho\frac{L}{S}$

$\rho = \rho_0(1 + \alpha(T - T_0))$