UNIVERSIDAD POLITECNICA DE VALENCIA

Third midterm exam FFI
January 8th 2016
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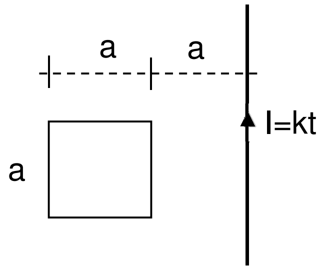
Applied Physics Dept.

1. (3 points) A square loop with side ***a*** and resistance ***R*** is placed at a distance ***a*** from an infinite and straight current carrying conductor; along such conductor flows a current ***I=kt*** (k constant and positive). At an instant t>0, compute:

a) Magnetic flux ϕ through the loop.
b) Induced electromotive force ε on the loop.
c) Intensity of current i flowing along the loop, clearly giving its direction.
d) Mutual inductance coefficient between conductor and loop.
e) Resulting horizontal force acting on loop.

1. (3 puntos) Una espira cuadrada de lado ***a*** y resistencia ***R*** se encuentra a una distancia ***a*** de un conductor rectilíneo e indefinido por el que circula una corriente ***I=kt*** (k constante y positivo). Para un instante t>0 calcular:

a) Flujo magnético ϕ que atraviesa la espira.
b) Fuerza electromotriz ε inducida en la espira.
c) Intensidad de corriente i que circula por la espira, indicando claramente su sentido.
d) Coeficiente de inducción mutua entre conductor y espira.
e) Fuerza resultante horizontal que actúa sobre la espira.



a) Magnetic field at a point P over the loop (distance x to the conductor) is $B=\frac{\mu_0 kt}{2\pi x}$ (perpendicular to the plane of paper sheet) and exiting from paper to us. Taking a vertical strip (sides a and dx), the magnetic flux through the loop is:

$$\phi=\int_{loop}\vec{B}\cdot d\vec{S}=\int_a^{2a}\frac{\mu_0 kt}{2\pi x}a\,dx=\frac{\mu_0 kta}{2\pi}\int_a^{2a}\frac{dx}{x}=\frac{\mu_0 kta}{2\pi}ln2$$

b) $|\varepsilon|=\frac{d\phi}{dt}=\frac{\mu_0 ka}{2\pi}ln2$

c) $i=\frac{\varepsilon}{R}=\frac{\mu_0 ka}{2\pi R}ln2$ As magnetic flux is exiting from paper to us, and it's increasing on time, the induced current should produce a magnetic flux (and then a magnetic field) opposite to this increasing. Induced current must flow in **clockwise direction** along the loop.

d) As we have already computed flux through the loop due to current I, mutual inductance coefficient between conductor and loop is: $M=\frac{\phi}{I}=\frac{\mu_0 kta}{2\pi kt}ln2=\frac{\mu_0 a}{2\pi}ln2$

e) The force acting on four sides of loop comes from equation $\vec{F}=i\vec{L}\times\vec{B}$, being i the induced current on loop. On upper and lower sides of loop, forces are vertical, equal and with opposite directions, being zero their resultant force. We'll only have horizontal forces acting on left and right side of the loop; along each side, the magnetic field is constant, being these forces (in modulus):

$F_{left\ side}=iLB_{x=2a}=(\frac{\mu_0 ka}{2\pi R}ln2)a\frac{\mu_0 kt}{2\pi(2a)}=(\frac{\mu_0 k}{2\pi})^2\frac{at}{2R}ln2$ pointing to right

$F_{right\ side}=iLB_{x=a}=(\frac{\mu_0 ka}{2\pi R}ln2)a\frac{\mu_0 kt}{2\pi a}=(\frac{\mu_0 k}{2\pi})^2\frac{at}{R}ln2$ pointing to left. So, the resulting horizontal force is:

$F_{horizontal}=F_{right\ side}-F_{left\ side}=(\frac{\mu_0 k}{2\pi})^2\frac{at}{R}ln2-(\frac{\mu_0 k}{2\pi})^2\frac{at}{2R}ln2=(\frac{\mu_0 k}{2\pi})^2\frac{at}{2R}ln2$ pointing **to left**.

2. (2 points) An inductor ***200 mH*** sized and a ***3 Ω*** resistor are connected in series, and flowed by an intensity of current increasing on time with a rate ***1000 A/s*** **($I=10^3t$ A).**

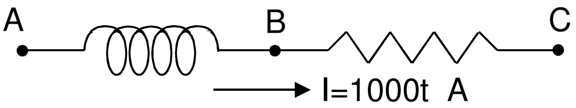
a) Calculate the induced electromotive force on terminals of inductor (terminals A and B).

b) At time ***t=0,1 s***, compute the total difference of potential between terminals of association (V_A-V_C).

2. (2 puntos) Una autoinducción de ***200 mH*** y una resistencia de ***3 Ω*** están conectadas en serie, y recorridas por una corriente que aumenta a razón de ***1000 A/s*** **($I=10^3t$ A).**

a) Calcula la fuerza electromotriz inducida en los bornes de la autoinducción (terminales A y B).

b) En el instante ***t=0,1 s***, calcula la diferencia de potencial total entre los terminales de la asociación (V_A-V_C).



a) Intensity is $I=10^3t$ A. So, $\frac{di}{dt}=10^3\ A/s$ Therefore, electromotive force on terminals of inductor is:

$$\varepsilon = L\frac{di}{dt} = 200\cdot 10^{-3}10^3 = 200\ V$$

b) The difference of potential between terminals A and B is not depending on time, being A the positive terminal, and B the negative one. So, $V_A - V_B = 200\ V$ for any time t.

The difference of potential on terminals of resistor at time t=0,1 s is: $V_B - V_C = I(t=0,1)R = 1000\cdot 0,1\cdot 3 = 300\ V$

And the total difference of potential at time t=0,1 s: $V_A - V_C = (V_A - V_B) + (V_B - V_C) = 200 + 300 = 500\ V$

3. (3 points) A ***2,5 mF*** sized capacitor and a resistor ***3 Ω*** sized are connected in series (dipole RC). Along the dipole is flowing a sinusoidal current ***i(t)=5cos(100t) A***. Compute:

a) Instantaneous voltage on resistor $u_R(t)$.

b) Instantaneous voltage on capacitor $u_c(t)$.

c) Impedance Z and phase lag φ of dipole.

d) The instantaneous voltage u(t) between terminals of dipole.

e) The maximum reactive power on capacitor.

Note: You can both use degrees or radians to solve this exercise.

3. (3 puntos) Un condensador de ***2,5 mF*** y una resistencia de ***3 Ω*** están conectadas en serie (dipolo RC). Por el dipolo circula una corriente senoidal ***i(t)=5cos(100t) A***. Calcular:

a) La tensión instantánea en la resistencia $u_R(t)$.

b) La tensión instantánea en el condensador $u_C(t)$.

c) La impedancia Z y el ángulo de desfase φ del dipolo.

d) La tensión instantánea u(t) entre los terminales del dipolo.

e) La potencia reactiva máxima en el condensador.

Nota: Podéis utilizar tanto grados sexagesimales como radianes para resolver este ejercicio.

a) On a resistor, phase lag between voltage and intensity is zero. So, $u_R(t) = 3\cdot 5cos(100t) = 15cos(100t)\,V$

b) $X_C = \frac{1}{C\omega} = \frac{1}{2,5\cdot 10^{-3}\cdot 100} = 4\,\Omega$ Phase lag on a capacitor is –π/2. Therefore

$$u_c(t) = 4\cdot 5cos\left(100t - \frac{\pi}{2}\right) = 20cos\left(100t - \frac{\pi}{2}\right)V$$

c) $Z = \sqrt{R^2 + (L\omega - 1/C\omega)^2} = \sqrt{3^2 + (-4)^2} = 5\,\Omega$ $\quad tg\varphi = \frac{-\frac{1}{C\omega}}{R} = \frac{-4}{3} = -1,33 \Rightarrow \varphi = -53,1º = -0,93\ rad$

d) $U_m = I_m Z = 5\cdot 5 = 25\,V$ $\quad \varphi_i=0$ and then $\varphi = -0,93\ rad = \varphi_u - \varphi_i = \varphi_u$ So, instantaneous voltage on dipole is: $u(t) = 25cos(100t - 0,93)\,V$

e) The maximum reactive power on a dipole is $\frac{U_m I_m}{2}sin\varphi$. On a capacitor, $sin\varphi = sin(-90º) = -1$ and the maximum reactive power (in absolute value) on capacitor will be: $RP_{max} = \frac{20\cdot 5}{2}1 = 50\,w$

4. (2 points) An extrinsic ***n type*** semiconductor is made up by Si doped with ***10^{17} Sb atoms/cm^3*** (Sb is a donor of e^-). If the intrinsic concentration of Si at ***300 K*** is ***n_i=1,5·10^{10} cm^{-3}*** a) compute the concentration of electrons and holes on such semiconductor at 300 K. b) Explain if the net electric charge of semiconductor is positive, negative, or neutral.	**4**. (2 puntos) Un semiconductor extrínseco ***tipo n*** está formado por Si dopado con ***10^{17} átomos de Sb/cm^3*** (Sb es un donador de e^-). Si la concentración intrínseca del Si a ***300 K*** es ***n_i=1,5·10^{10} cm^{-3}*** a) Calcular la concentración de electrones y huecos en dicho semiconductor a 300 K. b) Razona si la carga eléctrica neta del semiconductor es positiva, negativa, o neutra.

a) According mass action law $n \cdot p = n_i^2 \Rightarrow n \cdot p = 1{,}5^2 \cdot 10^{20}$

As doping of donor concentration (N_D) is very high compared with intrinsic concentration (n_i) (10^7>>>10^{10}) , we can write that $n \approx N_D = 10^{17}\ e^- / cm^3$. And $p = \frac{n_i^2}{n} = \frac{1{,}5^2 \cdot 10^{20}}{10^{17}} = 2{,}25 \cdot 10^3\ h / cm^3$

b) When an intrinsic semiconductor is doped, even if the it's doped with donor or acceptor atoms, the doping atoms are electrically neutral, with net charge zero. So, the electric charge of doped semiconductor will be neutral, with zero net charge.

FORM

Magnetic Forces $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$ $\quad d\vec{\mathbf{F}} = I\, \vec{\mathbf{dl}} \times \vec{\mathbf{B}}$ $\quad \vec{\mu} = N \cdot I \cdot \vec{\mathbf{S}}$ $\quad \vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$ $\quad V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$

Sources of magnetic field $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$ $\quad \mu_0 = 4\pi 10^{-7}\ (I.S.units)$ $\quad B = \frac{\mu_0 I}{2\pi x}$

$B = \frac{\mu_0 I}{2R}$ $\quad \oint_L \vec{\mathbf{B}} \cdot \vec{\mathbf{dl}} = \mu_0 \sum I$ $\quad B = \frac{\mu_0 N I}{l}$

Electromagnetic induction $|\varepsilon| = \frac{d\phi}{dt}$ $\quad \phi = L \cdot I$ $\quad \phi_{21} = M \cdot I_1$ $\quad \varepsilon = L\frac{di}{dt}$ $\quad W_L = \frac{1}{2} L \cdot I^2$

Alternating current $\varphi = \varphi_u - \varphi_i$ $\quad X_L = L\omega$ $\quad X_C = \frac{1}{C\omega}$ $\quad U_{rms} = \frac{U_m}{\sqrt{2}}$ $\quad I_{rms} = \frac{I_m}{\sqrt{2}}$

$$tg\varphi = \frac{L\omega - 1/C\omega}{R} \qquad Z = \frac{U_m}{I_m} = \sqrt{R^2 + (L\omega - 1/c\omega)^2}$$

$$P(t) = u(t) \cdot i(t) = U_m I_m \cos\varphi \sin^2 \omega t + \frac{U_m I_m}{2} \sin\varphi \sin\omega t \qquad P_{av} = \frac{U_m I_m}{2}\cos\varphi \qquad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Semiconductors $n \cdot p = n_i^2$ $\quad N_A + n = N_D + p$