UNIVERSIDAD POLITECNICA DE VALENCIA

Retaking First exam FFI
January 25th 2016
Year 2015/16

Applied Physics Dept.

1. (2,5 puntos) Dos cargas puntuales de **-2 μC** y **4 μC** se encuentran en el vacío en las posiciones **A(8,0) m** y **B(-4,0)** m respectivamente. Calcula:

a) El campo eléctrico resultante en el punto **C(2,0) m**. Indica el vector campo eléctrico que crea cada carga y el campo eléctrico resultante.

b) El trabajo realizado por el campo eléctrico al desplazar una carga de **-3 μC** desde el punto **C** hasta el **infinito**. ¿Quién realiza el trabajo?

1. (2,5 points) Given two point charges **-2 μC** and **4 μC**, on vacuum, placed at points **A(8,0) m** and **B(-4,0) m**, compute:

a) The electric field vector at point **C(2,0) m**. Give the electric field vector created by each charge and the resulting electric field vector.

b) The work done by the forces of the electric field to carry a **-3 μC** point charge from point **C** to **infinite**. ¿Who is doing this work?

a) The electric field due to each charge, according Coulomb's law is:

$$\vec{E}_C(-2)=k(\frac{2\cdot 10^{-6}}{6^2})\vec{i}=9\cdot 10^9\frac{2\cdot 10^{-6}}{36}\vec{i}=\frac{1}{2}10^3\vec{i}=500\vec{i}\ N/C$$

$$\vec{E}_C(4)=k(\frac{4\cdot 10^{-6}}{6^2})\vec{i}=9\cdot 10^9\frac{4\cdot 10^{-6}}{36}\vec{i}=10^3\vec{i}=1000\vec{i}\ N/C$$

And the resulting electric field at point C: $\vec{E}_C=\vec{E}_C(-2)+\vec{E}_C(4)=500\vec{i}+1000\vec{i}=1500\vec{i}\ N/C$

b) Wu must on first calculate the electric potential at point C:

$$V_C=k(\frac{-2\cdot 10^{-6}}{6}+\frac{4\cdot 10^{-6}}{6})=9\cdot 10^9\frac{2\cdot 10^{-6}}{6}=3\cdot 10^3=3000\ V$$

As $V_\infty=0$ $\quad W_{C\infty}=q(V_C-V_\infty)=-3\cdot 10^{-6}(3000-0)=-9\cdot 10^{-3}\ J=-9\ mJ$

As work is negative, it means that it's done against the forces of electric field by external forces.

2. (2,5 puntos) Entre los puntos ***A*** y ***B*** de la asociación de condensadores de la figura se aplica una diferencia de potencial **V**.

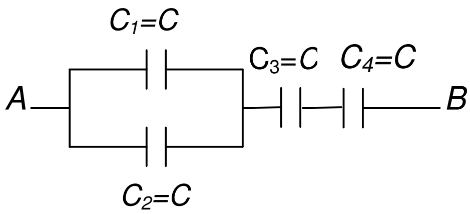
a) Calcula la capacidad equivalente de la asociación, la carga y la tensión en cada condensador.

b) Repite el punto anterior después de desconectar la batería entre **A** y **B** y de introducir un dieléctrico con $\varepsilon_r=2$ en el condensador **C₁**.

2. (2,5 points) Between points ***A*** y ***B*** of association of capacitors on picture is applied a difference of potential **V**.

a) Compute the equivalent capacitance of association, the charge and voltage on each capacitor.

b) Repeat the before point after disconnecting the battery between **A** and **B** and inserting a dielectric with $\varepsilon_r=2$ on capacitor **C₁**.



a) C_1 and asociated in parallel, and ist equivalent capacitor is associated in series with C_3 and C_4. Therefore:

$$\frac{1}{C_{eq}}=\frac{1}{2C}+\frac{1}{C}+\frac{1}{C}=\frac{5}{2C}\Rightarrow C_{eq}=\frac{2}{5}C$$

If V is the voltage applied between A and B, the charge taken by the equivalent capacitor would be:

$Q_{eq}=C_{eq}V=\frac{2}{5}CV$ But this equivalent charge is that taken by C_4, equal to that taken by C_3 because both capacitors are connected in series: $Q_3=Q_4=Q_{eq}=\frac{2}{5}CV$

As C_1 and C_2 are equal, both capacitors have the same charge, a half of that of C_3: $Q_1=Q_2=\frac{1}{5}CV$

And the potential on each capacitor can be calculated from capacitance and charge ($V=\frac{Q}{C}$):

$V_1 = V_2 = \frac{1}{5}CV \qquad V_3 = V_4 = \frac{2}{5}C \qquad$ Obviously, it's verified that $V_1 + V_3 + V_4 = V$

b) If a dielectric with $\varepsilon_r = 2$ is inserted inside C_1, the new capacitance of C_1 will be 2C, and the new equivalent capacitance is: $\frac{1}{C'_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} = \frac{7}{3C} \Rightarrow C'_{eq} = \frac{3}{7}C$ and

If V is disconnected, the charge on C_3 and C_4 will remain unaltered $Q'_3 = Q'_4 = \frac{2}{5}CV \Rightarrow V'_3 = V'_4 = \frac{2}{5}V$

As C_1 and C_2 are connected in parallel, their potentials are equal: $V'_1 = \frac{Q'_1}{2C} = V'_2 = \frac{Q'_2}{C} \Rightarrow Q'_1 = 2Q'_2$

and the addition of charges on C_1 plus C_2 will be equal to that of C_3: $Q'_1 + Q'_2 = Q'_3 = \frac{2}{5}CV \Rightarrow 3Q'_2 = \frac{2}{5}CV \Rightarrow$

$\Rightarrow Q'_2 = \frac{2}{15}CV$ and $Q'_1 = \frac{4}{15}CV$. So $V'_1 = V'_2 = \frac{2}{15}V$

The new difference of potential between A and B is: $V' = V'_1 + V'_3 + V'_4 = \frac{14}{15}V$

3. (2,5 puntos) Define resistencia equivalente de un conjunto de resistencias. Demuestra que la resistencia equivalente de una asociación de n resistencias en paralelo es $\frac{1}{R_{eq}} = \sum_1^n \frac{1}{R_i}$	**3.** (2,5 points) Define the equivalent resistance of a set of resistors. Demonstrate that the equivalent resistance of an association of n resistors in parallel is $\frac{1}{R_{eq}} = \sum_1^n \frac{1}{R_i}$

The equivalent resistor of a set of resistors is that resistor flowed by the same intensity of current that the set of resistors when subjected to the same difference of potential.

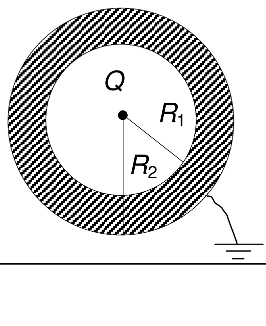
If we have n resistors associated in parallel, the equivalent resistance is that verifying: $I = \frac{V}{R_{eq}}$

on each resistor (i) is verified that $I_i = \frac{V_i}{R_i}$. But every resistors are subjected to the same potential: $V=V_i$

And the total intensity flowing along the set is the addition of the intensities flowing along each resistor:

$$I = \frac{V}{R_{eq}} = \sum_{i=1}^{n} I_i = \sum_{i=1}^{n} \frac{V_i}{R_i} = \sum_{i=1}^{n} \frac{V}{R_i} = V\sum_{i=1}^{n} \frac{1}{R_i} \Rightarrow \frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

4. (2,5 puntos) La figura muestra una esfera metálica hueca de radios interior y exterior $\boldsymbol{R_1}$ y $\boldsymbol{R_2}$, respectivamente. Dicha esfera se encuentra conectada a tierra. Se coloca una carga puntual positiva, $\boldsymbol{Q}$, en el centro de la esfera. a) ¿Cuál es la distribución de cargas en las superficies interior y exterior de la esfera? b) Obtén $\boldsymbol{E(r)}$ para $\boldsymbol{r< R_1, R_1< r< R_2, r> R_2}$. c) Obtén $\boldsymbol{V(r)}$ para $\boldsymbol{r< R_1, R_1\le r\le R_2, r> R_2}$. d) Si desconectamos la esfera metálica de tierra, ¿Cuál es la distribución de cargas en las superficies interior y exterior de la esfera? Justifica las respuestas	**4.** (2,5 points) The picture shows a hollow and conductor sphere (inner radius $\boldsymbol{R_1}$ and outer radius $\boldsymbol{R_2}$) linked to ground. There is a positive point charge **Q** at the centre of sphere. a) ¿Which is the distribution of charges on inner and outer surfaces of sphere? b) Get $\boldsymbol{E(r)}$ for $\boldsymbol{r< R_1, R_1< r< R_2, r> R_2}$. c) Get $\boldsymbol{V(r)}$ for $\boldsymbol{r< R_1, R_1\le r\le R_2, r> R_2}$. d) If we disconnect the conductor sphere from ground, ¿which is the distribution of charges on inner and outer surfaces of sphere? The answers must be justified



a) There is total influence between the charge and the inner surface of sphere. So, on inner surface of sphere is induced a charge –Q. As the sphere is connected to ground, on outer surface of sphere isn't induced any charge.
b) Applying Gaus's law to a sphere:

r<R$_1$: $\phi = \int_{sphere} \vec{E} \cdot d\vec{S} = \int E dS = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$

R$_1$< r< R$_2$, and ***r> R$_2$*** $\phi = E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0} \Rightarrow E = 0$

c) As potential at infinite is zero and the sphere is linked to ground (V=0):

R$_1$≤ r≤ R$_2$, r> R$_2$, and ***r> R$_2$*** V=0

r<R$_1$ We'll calculate the potential at a point inside the sphere (radius r) as the difference of potential between this point and another point with radius R_1 where the potential is zero:

$$V_r - V_{R_1} = V_r = \int_{sphere} \vec{E} \cdot d\vec{r} = \int E dr = \int_r^{R_1} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0} (-\frac{1}{r})_r^{R_1} = \frac{Q}{4\pi\varepsilon_0} (\frac{1}{r} - \frac{1}{R_1}) = \frac{Q}{4\pi\varepsilon_0 r} - \frac{Q}{4\pi\varepsilon_0 R_1}$$

d) If we disconnect the sphere from ground, the distribution of charges remains unaltered, because the system is already in electrostatic equilibrium.

Form

Electrostatics

$\vec{F} = K\frac{q_1 q_2}{r^2}\vec{u}_r$ $\quad \vec{E} = \frac{\vec{F}}{q}$ $\quad K = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 (S.I.)$ $\quad V_A - V_B = \int_A^B \vec{E} \cdot \vec{dr}$

$\vec{E} = K\frac{q}{r^2}\vec{u}_r$ $\quad V = K\frac{q}{r}$ $\quad \int_S \vec{E} \cdot \vec{dS} = \frac{\sum Q}{\varepsilon_0}$ $\quad W_{AB} = q(V_A - V_B)$

Conductors and capacitors $\quad E = \frac{\sigma}{\varepsilon_0}$ $\quad C = \frac{Q}{V}$ $\quad C = \frac{\varepsilon_0 S}{d}$

$C_{eq} = \sum C_i$ $\quad \frac{1}{C_{eq}} = \sum \frac{1}{C_i}$ $\quad E_d = \frac{E}{\varepsilon_r}$ $\quad C_d = \varepsilon_r C$ $\quad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$

Direct Current $\quad \vec{J} = n \cdot e \cdot \vec{v}_a$ $\quad \vec{J} = \sigma \cdot \vec{E}$ $\quad R = \frac{V_1 - V_2}{I}$ $\quad R = \rho\frac{L}{S}$

1. Si te examinas de un único parcial: Debes resolver los cuatro ejercicios de ese parcial.
2. Si te examinas de dos parciales: Debes resolver los ejercicios 1, 2 y 3 de cada uno de los dos parciales (en total 6 ejercicios). Los 6 ejercicios puntúan igual.
3. Si te examinas de los tres parciales: Debes resolver los ejercicios 1 y 2 de cada uno de los tres parciales (en total 6 ejercicios). Los 6 ejercicios puntúan igual.

1. If you are sitting only one midterm exam: You must solve the four exercises of this midterm exam.
2. If you are sitting two midterm exams: You must solve exercises 1, 2 and 3 of both midterm exams. (6 exercises as a whole). Every exercises weight equal.
3. If you are sitting three midterm exams: You must solve exercises 1 and 2 of three midterm exams, (6 exercises as a whole). Every exercises weight equal.