	First mid term FFI exam **October, 24th , 2016** Year 2016/17	Applied Physics Dept.

1. (3 points) Plane **XY** of a reference system (infinite plane) is charged with uniform surface density of charge **σ=1 μC/m²**. Over **Z** axis, at point **(0,0,1)** there is a **-10 μC** point charge. Compute: **a)** (0,6) Electric field due to both charges at point **P(0,0,3)**. Give the result as a vector. **b)** (0,6) Calculate the difference of potential between point **R(0,1,1)** and point **S(0,2,1)**. **c)** (0,6) Calculate the work needed to carry a **2 μC** charge from **R** to **S**. Is this work done by the forces of electric field or by external forces to the electric field? **d)** (0,6) Calculate the difference of potential between point **T(0,0,2)** and point **P(0,0,3)**. **e)** (0,6) Calculate the coordinates of a point **U**, whose **Z** coordinate is positive, where the total electric field was null.	**1.** (3 puntos) El plano **XY** de un sistema de referencia (plano infinito) se encuentra cargado con una densidad superficial de carga uniforme **σ=1 μC/m²**. Sobre el eje **Z**, en el punto **(0,0,1)** hay una carga puntual de **-10 μC**. Calcular: **a)** (0,6) El campo eléctrico debido a ambas cargas en el punto **P(0,0,3)**. Expresarlo como un vector. **b)** (0,6) Calcula la diferencia de potencial entre el punto **R(0,1,1)** y el punto **S(0,2,1)**. **c)** (0,6) Calcula el trabajo necesario para llevar una carga de **2 μC** desde **R** hasta **S**. Este trabajo ¿es hecho por las fuerzas del campo eléctrico, o por fuerzas externas a él? **d)** (0,6) Calcula la diferencia de potencial entre el punto **T(0,0,2)** y el punto **P(0,0,3)**. **e)** (0,6) Calcula las coordenadas de un punto **U**, cuya coordenada **Z** sea positiva, donde el campo eléctrico total sea nulo.

a) $\vec{E}_P=(\frac{\sigma}{2\varepsilon_0}-k\frac{Q}{r^2})\vec{k}=(\frac{1\cdot 10^{-6}}{2\cdot 8,85\cdot 10^{-12}}-9\cdot 10^9\frac{10\cdot 10^{-6}}{2^2})\vec{k}=(56,5-22,5)10^3\vec{k}=34,5\cdot 10^3\vec{k}\ V/m$

b) $V_R-V_S=k\frac{Q}{d}-k\frac{Q}{d'}=-9\cdot 10^9 10\cdot 10^{-6}(\frac{1}{1}-\frac{1}{2})=-45000\ V$

c) $W=q(V_R-V_S)=2\cdot 10^{-6}(-45000)=-90\cdot 10^{-3}\ J$ This work is done by external forces to the electric field.

d) $V_T-V_P=\frac{\sigma}{2\varepsilon_0}(3-2)+(k\frac{Q}{1}-k\frac{Q}{2})=\frac{1\cdot 10^{-6}}{2\cdot 8,85\cdot 10^{-12}}+(-9\cdot 10^9 10\cdot 10^{-6}\frac{1}{2}=(56,5-45)\cdot 10^3=11500\ V$

e) $\frac{\sigma}{2\varepsilon_0}=k\frac{|Q|}{d^2}\Rightarrow d=\sqrt{\frac{2kQ\varepsilon_0}{\sigma}}=\sqrt{\frac{2\cdot 10\cdot 10^{-6}}{4\cdot\pi\cdot 1\cdot 10^{-6}}}=\sqrt{1,592}=1,26\ m$ U(0;0;2,26) m

2. (2,5 points) A drop of water (conductor material) is spherical with radius **2 mm**, having a net charge **8 nC.** **a)** (0,6) Compute the surface density of charge of drop. **b)** (0,6) By applying Gaus's law, compute the electric field at points inside the drop (**r<2 mm**) and outside the drop (**r>2 mm**). **c)** (0,6) Compute the electrostatic potential of drop. The before drop is joined to another equal drop with the same charge, in such way that both make up a new spherical drop. **d)** (0,7) Compute the electrostatic potential of new drop.	**2.** (2,5 puntos) Una gota de agua (material conductor) es esférica de radio **2 mm**, y tiene una carga neta de **8 nC.** **a)** (0,6) Calcula la densidad superficial de carga de la gota. **b)** (0,6) Aplicando el teorema de Gauss, calcula el campo eléctrico en puntos interiores (**r<2 mm**) y exteriores (**r>2 mm**) a la gota. **c)** (0,6) Calcula el potencial electrostático de la gota. La gota anterior se junta con otra gota idéntica a la anterior y la misma carga, de modo que ambas forman una nueva gota esférica. **d)** (0,7) Calcula el potencial electrostático de la nueva gota.

a) $\sigma = \frac{q}{S} = \frac{8 \cdot 10^{-9}}{4\pi 4 \cdot 10^{-6}} = \frac{1}{2\pi} 10^{-3} = 0{,}16 \cdot 10^{-3}\ C/m^2$

b) r>2 mm $E 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2} = k\frac{q}{r^2} = 9 \cdot 10^9 \frac{8 \cdot 10^{-9}}{r^2} = \frac{72}{r^2}$

r<2 mm E=0

c) $V = \int_{2 \cdot 10^{-3}}^{\infty} \frac{72}{r^2} dr = -\frac{72}{r}\Big|_{2 \cdot 10^{-3}}^{\infty} = \frac{72}{2 \cdot 10^{-3}} = 36000\ V$

It can also by computed: $V = k\frac{q}{R} = 9 \cdot 10^9 \frac{8 \cdot 10^{-9}}{2 \cdot 10^{-3}} = 36000\ V$

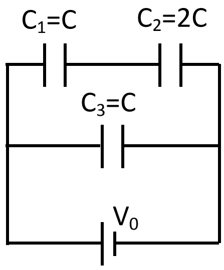
d) The radius R of new drop will be:

$2 \cdot \frac{4}{3}\pi (2 \cdot 10^{-3})^3 = \frac{4}{3}\pi R'^3 \Rightarrow R' = 2 \cdot 10^{-3} \sqrt[3]{2} = 2{,}52 \cdot 10^{-3}\ m$

And the charge of new drop: Q=8+8=16 nC

Therefore the electric potential of new drop: $V' = k\frac{Q}{R'} = 9 \cdot 10^9 \frac{16 \cdot 10^{-9}}{2{,}52 \cdot 10^{-3}} = 57140\ V$

3. (2,5 points) The association of capacitors on picture is connected to a difference of potential $\mathbf{V_0}$.	**3.** (2,5 puntos) La asociación de condensadores de la figura se conecta a una d.d.p. $\mathbf{V_0}$.
a) (0,7) Calculate the charge on each capacitor, $\mathbf{Q_1}$, $\mathbf{Q_2}$ and $\mathbf{Q_3}$.	**a)** (0,7) Calcula la carga en cada condensador, $\mathbf{Q_1}$, $\mathbf{Q_2}$ y $\mathbf{Q_3}$.
b) (0,6)Calculate the voltage on each capacitor, $\mathbf{V_1}$, $\mathbf{V_2}$ and $\mathbf{V_3}$.	**b)** (0,6) Calcula la tensión en cada condensador, $\mathbf{V_1}$, $\mathbf{V_2}$ y $\mathbf{V_3}$.
c) (0,6) A dielectric with relative dielectric permittivity $\boldsymbol{\varepsilon_r}$**=2** is introduced between the plates of $\mathbf{C_2}$. Compute the new charges on each capacitor.	**c)** (0,6) En $\mathbf{C_2}$ se introduce un dieléctrico de permitividad relativa $\boldsymbol{\varepsilon_r}$**=2**. Calcula las nuevas cargas en cada condensador.
d) (0,6) Capacitor $\mathbf{C_3}$ is removed. ¿What are now the new charges on $\mathbf{C_1}$ and $\mathbf{C_2}$?	**d)** (0,6) Se retira del conjunto el condensador $\mathbf{C_3}$. ¿Cuáles son ahora las nuevas cargas en $\mathbf{C_1}$ y $\mathbf{C_2}$?



a) C_1 and C_2 are asociated in series and then theirs charges are equal: $Q_1 = Q_2$ verifying that:

$$\frac{Q_1}{C} + \frac{Q_2}{2C} = V_0$$

From these equations: $Q_1 = Q_2 = \frac{2}{3} C V_0$

On the other hand, the voltage on C_3 is V_0 and so $Q_3 = C V_0$

b) Voltage on C_3 is $V_3 = V_0$.

On C_1: $V_1 = \frac{Q_1}{C} = \frac{2}{3} V_0$

On C_2: $V_2 = \frac{Q_2}{2C} = \frac{1}{3} V_0$

Obviously, $V_1 + V_2 = V_0$

c) After inserting the dielectric on C_2, its new capacitance is C'_2=4C and the new charges come from system:

$$Q'_1 = Q'_2 \qquad \frac{Q'_1}{C} + \frac{Q'_2}{4C} = V_0$$

It solution is: $Q'_1 = Q'_2 = \frac{4}{5} C V_0$

As the difference of potential on C_3 doesn't change, its charge remains equal: $Q'_3 = Q_3 = CV_0$

d) If C_3 is removed, the difference of potential on upper branch remains equal, and then the charges on C_1 and C_2:

$$Q''_1 = Q''_2 = \frac{4}{5} C V_0$$

4. (2 points) Given the association of resistors on figure, compute:

a) (0,8) $\mathbf{V_{AC}}$, **I** and $\mathbf{V_{AB}}$

The **4 Ω** resistor has been built with a wire of Nicromo (alloy of Ni and Cr). The wire had **2 mm²** of cross section and length **8 m**:

b) (0,7) Compute the density of current **J** on wire of resistor and the electric field **E** inside.

c) (0,5) Compute the conductivity **σ** of Nicromo.

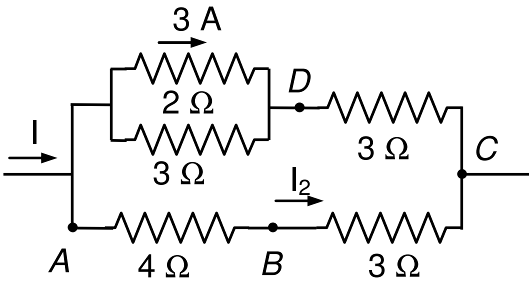
4. (2 puntos) En la asociación de resistencias de la figura, calcular:

a) (0,8) $\mathbf{V_{AC}}$, **I** y $\mathbf{V_{AB}}$

La resistencia **4 Ω** se ha fabricado con un hilo de Nicromo (aleación de Ni y Cr). El hilo tenía una sección de **2 mm²** y una longitud de **8 m**:

b) (0,7) Calcula la densidad de corriente **J** en el hilo de la resistencia y el campo eléctrico **E** en su interior.

c) (0,5) Calcula la conductividad **σ** del Nicromo.



a) $V_{AD} = 3 \cdot 2 = 6\,V \qquad I_{3\Omega} = \frac{6}{3} = 2\,A \qquad I_{DC} = 3 + 2 = 5\,A \qquad V_{DC} = 5 \cdot 3 = 15\,V$

$V_{AC} = V_{AD} + V_{DC} = 6 + 15 = 21\,V \qquad I_2 = \frac{21}{7} = 3\,A \qquad I = 5 + 3 = 8\,A \qquad V_{AB} = 3 \cdot 4 = 12\,V$

b) $J = \frac{I_2}{S} = \frac{3}{2 \cdot 10^{-6}} = 1{,}5 \cdot 10^6\ A/m^2 \qquad E = \frac{V}{d} = \frac{12}{8} = 1{,}5\,N/C$

c) $\sigma = \frac{1}{\rho} = \frac{L}{RS} = \frac{8}{4 \cdot 2 \cdot 10^{-6}} = 10^6\ S/m$

Form

Electrostatics

$\vec{F} = K \frac{q_1 q_2}{r^2} \vec{u}_r \qquad \vec{E} = \frac{\vec{F}}{q} \qquad K = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 (S.I.) \qquad V_A - V_B = \int_A^B \vec{E} \cdot \vec{dr}$

$\vec{E} = K \frac{q}{r^2} \vec{u}_r \qquad V = K \frac{q}{r} \qquad \oint_S \vec{E} \cdot \vec{dS} = \frac{\sum Q}{\varepsilon_0} \qquad W_{AB} = q(V_A - V_B)$

Conductors and capacitors $\qquad E = \frac{\sigma}{\varepsilon_0} \qquad C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 S}{d}$

$C_{eq} = \sum C_i \qquad \frac{1}{C_{eq}} = \sum \frac{1}{C_i} \qquad E_d = \frac{E}{\varepsilon_r} \qquad C_d = \varepsilon_r C \qquad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$

Direct Current $\qquad \vec{J} = n \cdot e \cdot \vec{v}_a \qquad \vec{J} = \sigma \cdot \vec{E} \qquad R = \frac{V_1 - V_2}{I} \qquad R = \rho \frac{L}{S}$

$\rho = \rho_0 (1 + \alpha (T - T_0)) \qquad P_R = IV = I^2 R = \frac{V^2}{R}$