UNIVERSIDAD POLITECNICA DE VALENCIA

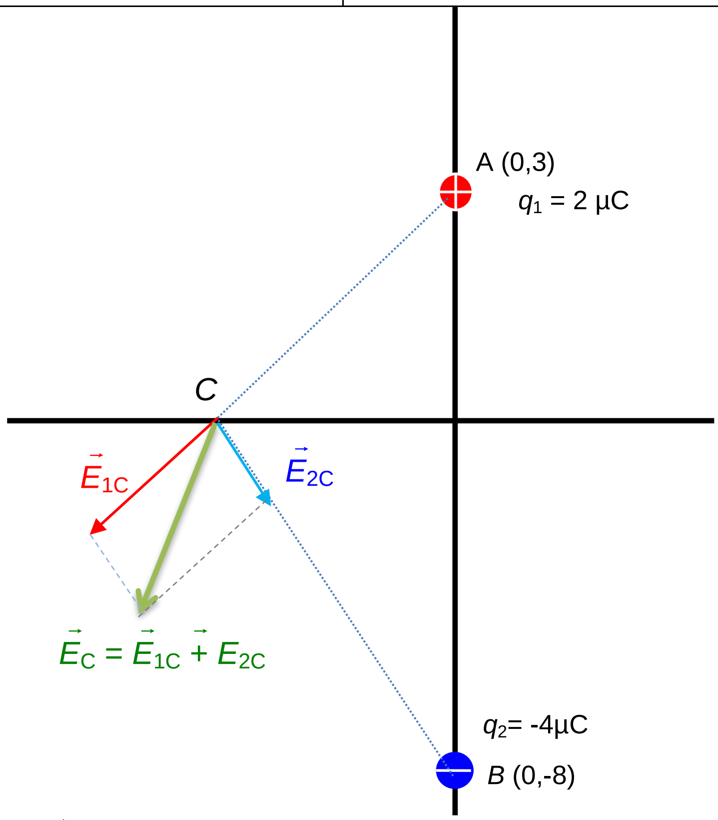
Retaking First exam FFI
Recuperación 1er parcial FFI
January 27th 2017
Year-Curso 2016/17

Applied Physics Dept.
Dep. de Física Aplicada

1. (2,5) Two point charges of 2 μC and -4μ C are placed on vacuum at points A(0,3) m and B(0,-8) m.
a) (1) Find the total electric field at point C(-4,0) m. Draw this total electric field and also the electric field vector created by each charge.
b) (0,8) Compute the work done by the electric field to move a -3 μC point charge from the infinite to point C. ¿Who's doing this work (the field or some force external to the field)?
c) (0,7) Find a point lying on Y axis between points A and B where the electric potential was zero.

1. (2,5) Dos cargas puntuales de 2μC y -4μC se encuentran en el vacío en las posiciones A(0,3) m y B(0,-8) m respectivamente.
a) (1) Calcula el campo eléctrico resultante en el punto C(-4,0) m. Dibuja los vectores campo eléctrico que crea cada carga y el campo eléctrico resultante.
b) 0,8) Calcula el trabajo realizado por el campo eléctrico al desplazar una carga de -3μC desde el infinito al punto C. ¿Quién realiza el trabajo?
c) (0,7) Halla un punto entre A y B (en el eje OY) en el que el potencial sea cero.

a)



0,3 points

$$\vec{E}_{1C} = K\frac{q_1}{r_1^2}\vec{u}_{r1} = 9\cdot10^9\frac{2\cdot10^{-6}}{5^2}\frac{\left(-4\vec{i}-3\vec{j}\right)}{5} = \left(-576\vec{i}-432\vec{j}\right)(\text{N/C})$$

0.6 points

$$\vec{E}_{2C} = K\frac{q_2}{r_2^2}\vec{u}_{r2} = 9\cdot10^9\frac{4\cdot10^{-6}}{80}\left(\frac{4\vec{i}-8\vec{j}}{\sqrt{80}}\right) = \left(201{,}25\vec{i}-402{,}4\vec{j}\right)(\text{N/C})$$

0.1 points

$$\vec{E}_C = \vec{E}_{1C} + \vec{E}_{2C} = \left(-374{,}8\vec{i}-834{,}4\vec{j}\right)\ (\text{N/C})$$

b) $V_{1C} = K\frac{q_1}{r_1} = 9\cdot10^9\frac{2\cdot10^{-6}}{5} = 3600\ \text{V}$

$$V_{2C} = K\frac{q_2}{r_2} = 9\cdot10^9\frac{-4\cdot10^{-6}}{\sqrt{80}} = -4024{,}9\ \text{V}$$

$$V_C = V_{1C} + V_{2C} = -424{,}9\ \text{V}$$

0,4 points

$W_{\infty C} = q(V_\infty - V_C) = -3\cdot 10^{-6}(0-(-424{,}9)) = -1{,}274\cdot 10^{-3}\ \text{J}$ **0,2 points**

The negative sign is telling us that the work to carry the charge from infinite to point C is done by external forces. **0,2 points**

c) We are looking for a point P in such way that

$V_P = V_{1P} + V_{2P} = 0\ \text{V}$ being $V_P = k\dfrac{q_1}{r_{1P}} + k\dfrac{q_2}{r_{2P}} = 0\,V$

Let we call y the distance of such point to point A. Therefore:

$\dfrac{2}{y} = \dfrac{4}{(11-y)} \Rightarrow y = 3{,}66\ m$ then, point P is P(0,-0,66) m **0,7 points**

2. (2,5) State Gaus's law (1,2) and apply it to calculate the electric field created by an spherical conductor charged with a charge Q:

a) (0,7) At a distance *2R* from its centre
b) (0,6) At a distance *R/2* from its centre

2. (2,5) Enuncia el teorema de Gauss (1,2) y aplícalo para calcular el campo eléctrico creado por un conductor esférico cargado con una carga Q:

a) (0,7) A una distancia *2R* de su centro
b) (0,6) A una distancia *R/2* de su centro

Gauss's law:
The flux of the electric field through a closed surface *S* equals the enclosed charge inside such surface divided into the dielectric permittivity. **1,2 points**

a) On first we choose a spherical surface with radius 2R, concentric with the charged surface. At any point of such surface electric field and surface vector are parallel and the modulus of electric field is constant. Therefore:

$$\Phi = \oint_S \vec{E}\cdot d\vec{S} = \oint_S E\cdot dS = E\oint_S dS = E\cdot 4\pi(2R)^2 = \frac{Q}{\varepsilon_0} \quad \Rightarrow\ E = \frac{Q}{16\pi\varepsilon_0 R^2}$$ **0,7 points**

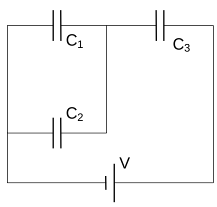
b) As we have done above:

$$\Phi = \oint_S \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

Now, as we are inside a charged conductor in equilibrium, the volumetric density of charge is null, there is no charge inside the conductor, and every charges are over its Surface. So, te flux of te electric field through the spherical surface of radius *R/2* is null and also the electric field. **0,6 points**

3. (2,5) The picture shows 3 equal capacitors with capacitance ***C*** each, connected to a difference of potential ***V***.
a) (1,2) Find the charge on each capacitor and the difference of potential on terminals of each one.
b) (1,3) The battery is removed and a dielectric with relative permittivity ε_r=4 is introduced on capacitor 1. Find the new charge on each capacitor and the difference of potential on terminals of each one.

3. (2,5) La figura muestra 3 condensadores iguales de capacidad ***C***, conectados a una diferencia de potencial ***V***.
a) (1,2) Halla la carga en cada condensador y la d.d.p. en bornes de cada condensador.
b) (1,3) Se retira la fuente y se introduce un dieléctrico de permitividad relativa ε_r=4 en el condensador 1. Halla la nueva carga en cada condensador y la d.d.p. en bornes de cada condensador.

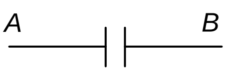


a) The equivalent capacitance of capacitors C_1 and C_2 in parallel is

$$C_{1,2} = \sum_{i=1}^{n} C_i = C_1 + C_2 = C + C = 2C$$

Capacitors $C_{1,2}$ and C_3 are connected in series; so, te equivalent capacitance of this set is:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n}\frac{1}{C_i} = \frac{1}{C_{1,2}} + \frac{1}{C_3} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C} \Rightarrow C_{eq} = \frac{2C}{3}$$



$C_{eq} = 2C/3$

The total charge of system when a difference of potential *V* is applied is:

$$Q_T = C_{eq}V = \frac{2}{3}CV$$ **0,2 points**

Both capacitors C_3 y $C_{1,2}$ have equal charges, it is: $Q_T = Q_3 = Q_{1,2} = \frac{2}{3}CV$

$Q_T = Q_1 + Q_2 = \frac{2}{3}CV$ and $Q_1 = Q_2 = \frac{1}{3}CV$ **0,5 points**

$$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_{1,2}}{C_{1,2}} = \frac{1}{3}V$$

$$V_3 = \frac{Q_3}{C_3} = \frac{2}{3}V$$

0,5 points

b) When the battery is disconnected, the total charge of set remains constant, and when the dielectric is introduced on C_1, the new equivalent capacitance is:

$$C_{1,2} = \sum_{i=1}^{n} C_i = C_1 + C_2 = 4C + C = 5C$$

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n}\frac{1}{C_i} = \frac{1}{C_{1,2}} + \frac{1}{C_3} = \frac{1}{5C} + \frac{1}{C} = \frac{6}{5C} \qquad \Rightarrow \qquad C_{eq} = \frac{5}{6}C$$ **0,1 points**

$Q_T = \frac{2}{3}CV \qquad Q_T = Q_3 = Q_{1,2} = \frac{2}{3}CV$

$Q_T = Q_1 + Q_2 = \frac{2}{3}CV$ **0,2 points**

Capacitors 1 and 2 are connected in parallel to the same d.d.p. Therefore:

$\frac{Q_1}{4C_1} = \frac{Q_2}{C_2} \qquad \frac{Q_!}{4C} = \frac{Q_2}{C}$, and $Q_1 = 2Q_2 \qquad Q_1 = \frac{8}{15}CV \qquad Q_2 = \frac{2}{15}CV$ **0,5 points**

Voltages on terminals of each capacitor will be:

$$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_{1,2}}{C_{1,2}} = \frac{2}{15}V$$

$$V_3 = \frac{Q_3}{C_3} = \frac{2}{3}V$$

0,5 points

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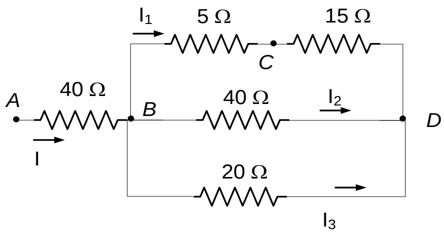
4. (2,5) On the association of resistors of the picture, is known that V_{BD} = **4 V.**
Compute:
a) (1,5) I_1, I_2, I_3, I, V_{CD}, V_{BC}, V_{AB} and V_{AD}.
b) (0,5) Equivalent resistance between points A and D
c) (0,5) Equivalent resistance between points A and C

4. (2,5) En la asociación de resistencias de la figura, se sabe que V_{BD} = **4 V.**
Calcula:
a) (1,5) I_1, I_2, I_3, I, V_{CD}, V_{BC}, V_{AB} y V_{AD}.
b) (0,5) Resistencia equivalente entre A y D
c) (0,5) Resistencia equivalente entre A y C



By applying Ohm's law to 40 Ω resistor:

$$V_{BD} = 4 = I_2 40 \quad \Rightarrow \quad I_2 = \frac{4}{40} = 0{,}1\ A$$

$$V_{BD} = 4 = I_3 20 \quad \Rightarrow \quad I_3 = \frac{4}{20} = 0{,}2\ A$$

$$V_{BD} = 4 = I_1 20 \Rightarrow I_1 = \frac{4}{20} = 0{,}2\ A$$

$I = I_1 + I_2 + I_3 = 0{,}5\ A$ **0,8 points**

So $V_{BC} = 5 \cdot I_1 = 5 \cdot 0{,}2 = 1V$

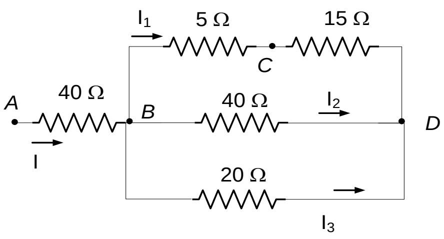
$V_{CD} = 15 \cdot 0{,}2 = 3V$

$V_{AB} = 5 \cdot I_1 = 40 \cdot 0{,}5 = 20\ V$

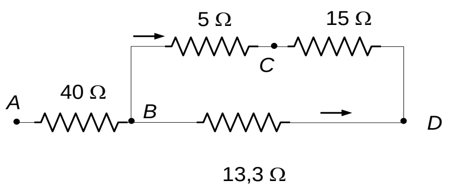
$V_{AD} = V_{AB} + V_{BD} = 20 + 4 = 24\,V$ **0,7 points**

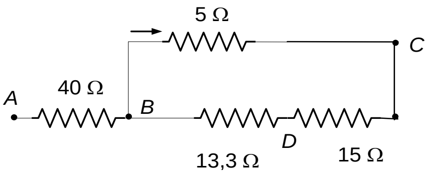
Equivalent resistance between A and D $R_{eqAD} = 40 + \left(\frac{1}{20} + \frac{1}{40} + \frac{1}{20}\right)^{-1} = 40 + 8 = 48\ \Omega$ **0,5 points**

Equivalent resistance between A and C $R_{eqAC} = 40 + 4{,}31 = 44{,}31\ \Omega$ **0,5 points**

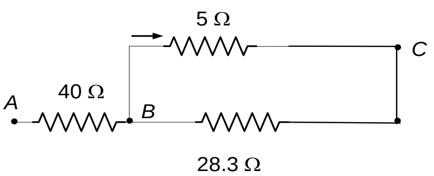


$$R_{40,20} = \left(\frac{1}{20} + \frac{1}{40}\right)^{-1} = 13{,}3\ \Omega$$

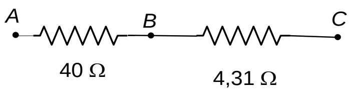




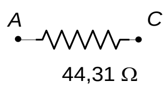
$$R_{13,3-15} = 13{,}30 + 15 = 28{,}3\Omega$$



$$R_{5,28,3} = \left(\frac{1}{5} + \frac{1}{28{,}3}\right)^{-1} = 4{,}31\ \Omega$$



$$R_{eqAC} = 40 + 4{,}31 = 44{,}31\ \Omega$$



Si te examinas sólo de una parte, debes resolver los cuatro problemas de esa parte.
If you are sitting only one part of the exam, then you have to solve the four problems of that part.

Si te examinas de dos partes, debes resolver los tres primeros problemas de ambas partes.
If you are sitting two parts of the exam, then you have to solve the three first problems of both parts.

Si te examinas de las tres partes, debes resolver los dos primeros problemas de cada una de las partes.
If you are sitting the three parts of the exam, then you have to solve the two first problems of every part.

Form on rear side – Fórmulas en la parte de atrás

Form - Fórmulas

Electrostatics

$\vec{F} = K\frac{q_1 q_2}{r^2}\vec{u}_r$ $\vec{E} = \frac{\vec{F}}{q}$ $K = \frac{1}{4\pi\varepsilon_0} = 9\cdot 10^9\,(S.I.)$ $V_A - V_B = \int_A^B \vec{E}\cdot\vec{dr}$

$\vec{E} = K\frac{q}{r^2}\vec{u}_r$ $V = K\frac{q}{r}$ $\oint_S \vec{E}\cdot\vec{dS} = \frac{\sum Q}{\varepsilon_0}$ $W_{AB} = q(V_A - V_B)$

Conductors and capacitors

$$E = \frac{\sigma}{\varepsilon_0} \qquad C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 S}{d}$$

$$C_{eq} = \sum C_i \qquad \frac{1}{C_{eq}} = \sum \frac{1}{C_i} \qquad E_d = \frac{E}{\varepsilon_r} \qquad C_d = \varepsilon_r C \qquad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$$

Direct Current

$$\vec{J} = n \cdot e \cdot \vec{v}_a \qquad \vec{J} = \sigma \cdot \vec{E} \qquad R = \frac{V_1 - V_2}{I} \qquad R = \rho \frac{L}{S}$$

$$\rho = \rho_0 (1 + \alpha (T - T_0)) \qquad P_R = IV = I^2 R = \frac{V^2}{R}$$