

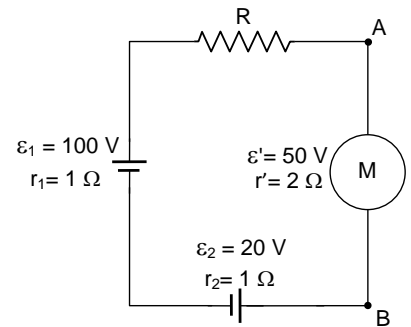


1. (2,5) On circuit on picture, the difference of potential between points A and B is **54 V** ($V_A - V_B = 54 \text{ V}$). Compute:

- (0,6) The intensity of current on circuit.
- (0,7) The resistance R.
- (0,7) Generated power and consumed power on every devices of circuit, clearly saying which of them act as generators and which act as receptors.
- (0,5) The efficiency of motor.

1. (2,5) En el circuito de la figura, la diferencia de potencial entre los puntos A y B es de **54 V** ($V_A - V_B = 54 \text{ V}$). Calcula:

- (0,6) La intensidad de corriente en el circuito.
- (0,7) La resistencia R.
- (0,7) Potencia generada y consumida en cada uno de los elementos del circuito, indicando claramente qué elementos actúan como generadores y cuáles como receptores.
- (0,5) El rendimiento del motor.



a) As $V_A - V_B > 0$, potential of A is higher than potential of B. So, the difference of potential between terminals of motor is:

$$V_A - V_B = \varepsilon' + Ir' = 50 + 2I = 54 \quad \Rightarrow \quad I = \frac{54 - 50}{2} = 2 \text{ A} \quad \text{Clockwise direction} \quad \mathbf{0,7 \text{ points}}$$

b) If we follow the other path from A to B:

$$V_A - V_B = -2(R + 1 + 1) - (-100 + 20) = 54 \quad \Rightarrow \quad R = 11 \Omega$$

Or also from Pouillet's law, taking in account that $I = \frac{\sum \varepsilon}{\sum R}$

$$I = \frac{\sum \varepsilon}{\sum R} = \frac{100 - 20 - 50}{1 + 1 + 2 + R} = \frac{30}{4 + R} = 2 \text{ A} \quad \Rightarrow \quad R = 11 \Omega \quad \mathbf{0,8 \text{ points}}$$

c) According to the direction of intensity, generator 1 acts as a generator, but generator 2 acts as a receptor:

$$\text{Generator 1: } P_g = \varepsilon_1 I = 100 \cdot 2 = 200 \text{ W} \quad P_{r_1} = I^2 r_1 = 2^2 \cdot 1 = 4 \text{ W} \quad P_s = P_g - P_{r_1} = 200 - 4 = 196 \text{ W}$$

$$\text{Generator 2: } P_{t_2} = \varepsilon_2 I = 20 \cdot 2 = 40 \text{ W} \quad P_{r_2} = I^2 r_2 = 2^2 \cdot 1 = 4 \text{ W} \quad P_{c_2} = P_{t_2} + P_{r_2} = 40 + 4 = 44 \text{ W}$$

$$\text{Motor: } P_t = \varepsilon' I = 50 \cdot 2 = 100 \text{ W} \quad P_{r'} = I^2 r' = 2^2 \cdot 2 = 8 \text{ W} \quad P_c = P_t + P_{r'} = 100 + 8 = 108 \text{ W}$$

$$\text{Resistor: } P_R = I^2 R = 2^2 \cdot 11 = 44 \text{ W}$$

Obviously, the power supplied by all the generators equals that consumed by the rest of the circuit:

$$196 = 44 + 108 + 44 = 196 \quad \mathbf{0,8 \text{ points}}$$

d) The efficiency of motor is: $\eta_m = \frac{P_t}{P_c} = \frac{100}{108} = 0,926 \approx 93\%$

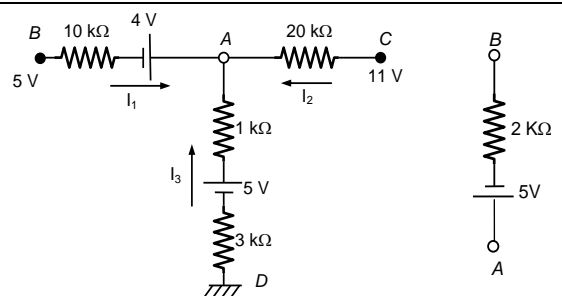
0,7 points

2. (2,5) Given the circuit on picture, compute:

- (1) The intensities flowing along each branch, by applying Kirchhoff's rules.
- (0,8) Thevenin's equivalent generator between A and B, clearly giving its polarity.
- (0,7) If the branch on the right is connected between A and B, find out if the 5 V generator is consuming or generating power,

2. (2,5) Dado el circuito de la figura, calcula:

- (1) Las intensidades que circulan por cada una de las ramas utilizando las leyes de Kirchhoff.
- (0,8) El generador equivalente de Thevenin entre B y A, indicando claramente su polaridad.
- (0,7) Si se conecta la rama de la derecha a A y B, indica si el elemento de 5 V consume o genera potencia y calcula su valor.



calculating this power.

a) Applying Kirchhoff's rules:

Junction rule: $\Sigma I = 0 \quad I_1 + I_2 + I_3 = 0$

Loop rule: $V_B - V_D = 5 = 10I_1 - 4 - 4I_3 + 5$

$V_C - V_D = 11 = 20I_2 - 4I_3 + 5$

0,7 points

By solving: $I_1 = 0,225 \text{ mA} \quad I_2 = 0,212 \text{ mA} \quad I_3 = -0,437 \text{ mA}$

0,3 points

b) Equivalent resistance between A and B:

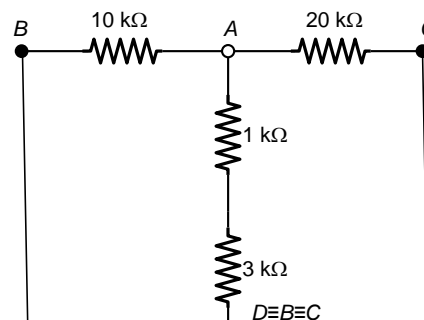
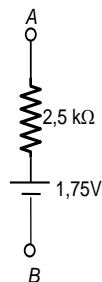
$$\frac{1}{4} + \frac{1}{20} + \frac{1}{10} = \frac{1}{R_{eq}}; \quad R_{eq} = 2,5 \text{ k}\Omega$$

Thèvenin's generator between A and B:

$$\varepsilon_T = V_A - V_B = -10I_1 + 4 = 1,75 \text{ V}$$

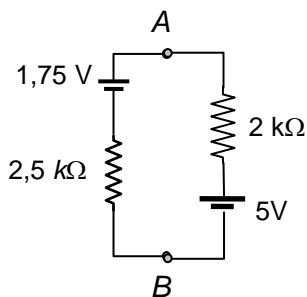
$$R_{eq} = 2,5 \text{ k}\Omega$$

Positive terminal of Thevenin's equivalent generator connected to A.



0,8 points

c)



If we add the new branch:

$$I = \frac{\Sigma \varepsilon}{\Sigma R} = \frac{5 - 1,75}{2,5 + 2} = 0,722 \text{ mA} \quad \text{Counterclockwise}$$

The 5 V generator acts as a generator, thus being its generated power:

$$P_g = \varepsilon I = 5 \cdot 0,722 = 3,61 \text{ mW}$$

0.7 points

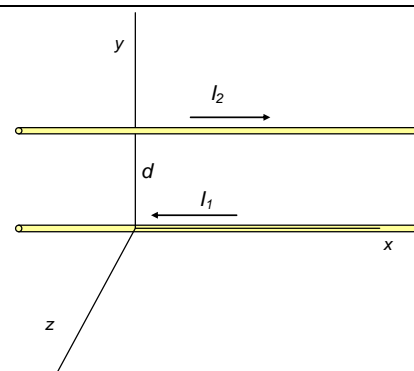
3. (2,5) Two infinite and straight conductors are placed on plane XY at a distance d from each other. They are flowed by two intensities with opposite directions $I_1=2I$ and $I_2=I$.

Compute the total magnetic field at points:

- a) (0,9) (0,d/2,0)
- b) (0,8) (0,2d,0)
- c) (0,8) (0,-2d,0)

3. (2,5) Dados dos conductores rectilíneos indefinidos situados en el plano XY por los que circulan dos corrientes de sentido contrario de intensidad $I_1=2I$ e $I_2=I$, separadas una distancia d como indica la figura. Calcula el campo magnético en los puntos:

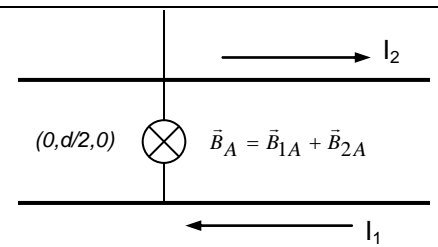
- a) (0,9) (0,d/2,0)
- b) (0,8) (0,2d,0)
- c) (0,8) (0,-2d,0)



a)

$$\vec{B}_A = \vec{B}_{1A} + \vec{B}_{2A} = \frac{\mu_0 I_1}{2\pi d/2} (-\vec{k}) + \frac{\mu_0 I_2}{2\pi d/2} (-\vec{k}) = \frac{\mu_0 2I}{2\pi d/2} (-\vec{k}) + \frac{\mu_0 I}{2\pi d/2} (-\vec{k}) = \frac{3\mu_0 I}{\pi d} (-\vec{k})$$

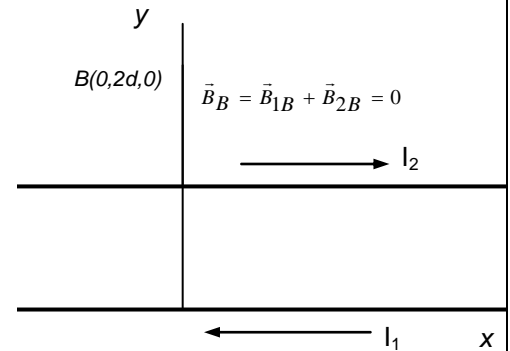
(0,9 points)



b)

$$\vec{B} = \vec{B}_{1B} + \vec{B}_{2B} = \frac{\mu_0 2I}{2\pi 2d} (-\vec{k}) + \frac{\mu_0 I}{2\pi d} (\vec{k}) = 0$$

(0,8 points)



c)
$$\vec{B} = \vec{B}_{1C} + \vec{B}_{2C} = \frac{\mu_0 2I}{2\pi 2d} (\vec{k}) - \frac{\mu_0 I}{2\pi 3d} (\vec{k}) = \frac{\mu_0 I}{3\pi d} (\vec{k})$$

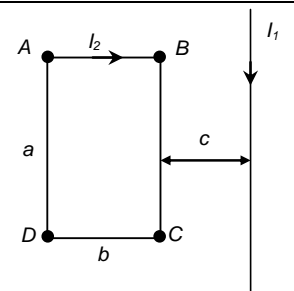
(0,8 points)

4. (2,5) Along the infinite straight carrying current wire flows an intensity I_1 . Along a rectangular loop with sides a and b placed in the same plane than that of wire flows a current I_2 . Compute:

- a) (0,7) the magnetic force on side CD by only taking in account the magnetic field created by I_1
- b) (0,7) the magnetic force on side AD by only taking in account the magnetic field created by I_1
- c) (0,3) the magnetic moment of loop
- d) (0,8) the flux of magnetic field created by I_1 through the rectangular loop.

4. (2,5) Sea un conductor rectilíneo infinito por el que circula una corriente de intensidad I_1 . Por una espira rectangular de lados a y b situada en el plano del conductor tal como se muestra en la figura circula una corriente de intensidad I_2 . Calcula:

- a) (0,7) la fuerza magnética sobre el lado CD debida sólo al campo magnético creado por I_1
- b) (0,7) la fuerza magnética sobre el lado AD debida sólo al campo magnético creado por I_1
- c) (0,3) el momento magnético de la espira
- d) (0,8) el flujo del campo magnético creado por I_1 a través de la espira rectangular.



a) We'll take a reference system with the X axis horizontal and positive direction to right, Y axis vertical and pointing to up, and Z axis exiting from paper sheet. Working on this reference system:

Magnetic field created by I_1 at a point inside the loop placed at a distance x from the wire $\vec{B} = \frac{\mu_0 I_1}{2\pi x} (-\vec{k})$

$$\vec{F}_{CD} = I_2 \int (d\vec{l} \times \vec{B}) = I_2 \int_c^{c+b} (dx(-\vec{i}) \times \frac{\mu_0 I_1}{2\pi x} (-\vec{k})) = I_2 \int_c^{c+b} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -dx & 0 & 0 \\ 0 & 0 & -\frac{\mu_0 I_1}{2\pi x} \end{vmatrix} = \frac{\mu_0 I_1 I_2}{2\pi} \int_c^{c+b} \frac{dx}{x} (-\vec{j}) = \frac{\mu_0 I_1 I_2}{2\pi} \ln \frac{c+b}{c} (-\vec{j}) \quad \text{0,7 points}$$

b) The magnetic field created by I_1 at a point between A and D is $\vec{B} = \frac{\mu_0 I_1}{2\pi(c+b)} (-\vec{k})$

And
$$\vec{F}_{AD} = I_2 \int (d\vec{l} \times \vec{B}) = I_2 \int (d\vec{l}) \times \vec{B} = I_2 (\vec{l} \times \vec{B}) = I_2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & a & 0 \\ 0 & 0 & -\frac{\mu_0 I_1}{2\pi(c+b)} \end{vmatrix} = \frac{\mu_0 I_1 I_2 a}{2\pi(c+b)} (-\vec{i}) \quad \text{0,7 points}$$

c) $\vec{m} = I_2 \vec{S} = I_2 ab (-\vec{k}) \quad \text{0,3 points}$

$$d) \quad d\Phi = \vec{B} \cdot d\vec{S} = B(x)adx = \frac{\mu_0 I}{2\pi x} adx \quad \phi = \int \vec{B} \cdot d\vec{S} = \int_c^{c+b} \frac{\mu_0 I a}{2\pi x} adx = \frac{\mu_0 I a}{2\pi} \int_c^{c+b} \frac{dx}{x} = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{c+b}{c}\right) \quad \mathbf{0,8 \text{ points}}$$

Si te examinas sólo de una parte, debes resolver los cuatro problemas de esa parte.

If you are sitting only one part of the exam, then you have to solve the four problems of that part.

Si te examinas de dos partes, debes resolver los tres primeros problemas de ambas partes.

If you are sitting two parts of the exam, then you have to solve the three first problems of both parts.

Si te examinas de las tres partes, debes resolver los dos primeros problemas de cada una de las partes.

If you are sitting the three parts of the exam, then you have to solve the two first problems of every part.

FORM – FÓRMULAS

Direct current $V_A - V_B = I \sum R - \sum \mathcal{E}$ $I = \frac{\sum \mathcal{E}}{\sum R}$ $P = V \cdot I$ $\mathcal{E} = \frac{dW}{dq}$ $P_R = I^2 \cdot R$ $P_g = \mathcal{E} \cdot I$

$P_t = \mathcal{E}' \cdot I$ $P_g - P_r = P_s$ $P_t + P_r = P_c$ $\eta_g = \frac{P_s}{P_g}$ $\eta_r = \frac{P_t}{P_c}$

Magnetic Forces $\vec{F} = q(\vec{v} \times \vec{B})$ $d\vec{F} = Id\vec{l} \times \vec{B}$ $\vec{\mu} = N \cdot I \cdot \vec{S}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$ $V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$

Sources of magnetic field $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$ $\mu_0 = 4\pi 10^{-7} \text{ (I.S.units)}$ $B = \frac{\mu_0 I}{2\pi x}$

$B = \frac{\mu_0 I}{2R}$ $\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I$ $B = \frac{\mu_0 NI}{l}$