First mid term FFI exam
October, 23th , 2017
Year 2017/18

Applied Physics Dept.

1. (3 points) Given the **three point charges** on picture:

a) Compute the **electric field** vector at point **P** due to the three charges. Draw the electric field created by each charge at P and the total electric field.

b) Compute the **work** needed to carry a **2 μC** charge **from point P** up to the **origin of coordinates**. Is this work done by the forces of the electric field or against them?

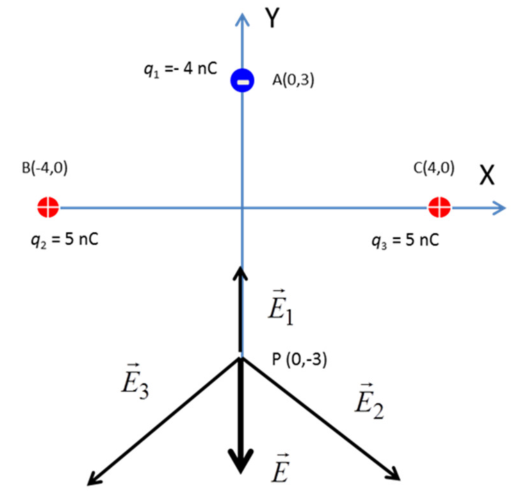
c) ¿What **point of the Y axis** should the charge $\mathbf{q_1}$ moved to (q_2 and q_3 remain unmoved) in order the **potencial** at the **origin** of coordinates was **null**?

1. (3 puntos) Dadas las **tres cargas** puntuales de la figura:

a) Calcula el vector **campo eléctrico** en el punto **P** debido a las tres cargas. Dibuja el campo que crea cada carga en P y el campo resultante.

b) Calcula el **trabajo** necesario para llevar una carga de **2 μC desde el punto P** hasta el **origen de coordenadas**. Este trabajo ¿es hecho por las fuerzas del campo, o en contra de ellas?

c) ¿A qué **punto del eje Y** deberíamos mover la carga $\mathbf{q_1}$ (q_2 y q_3 permanecen en los mismos puntos) para que el **potencial** en el **origen** de coordenadas **se anule**?



a) $\vec{E}_1 = k\frac{q_1}{r^2}\vec{j} = \frac{9\cdot10^9\cdot4\cdot10^{-9}}{6^2}\vec{j} = \vec{j}\ N/C$

$\vec{E}_2 = k\frac{q_2}{r^2}(\frac{4\vec{i}-3\vec{j}}{5}) = \frac{9\cdot10^9\cdot5\cdot10^{-9}}{5^2}(\frac{4\vec{i}-3\vec{j}}{5}) = \frac{9}{25}(4\vec{i}-3\vec{j})\ N/C$ $\qquad \vec{E}_3 = \frac{9}{25}(-4\vec{i}-3\vec{j})\ N/C$

$\vec{E} = \vec{E}_1+\vec{E}_2+\vec{E}_3 = \vec{j}+\frac{9}{25}(4\vec{i}-3\vec{j})+\frac{9}{25}(-4\vec{i}-3\vec{j}) = \vec{j}-2\frac{27}{25}\vec{j} = -\frac{29}{25}\vec{j}\ N/C$

b) $V_{P1} = k\frac{q_1}{d_1} = \frac{9\cdot10^9(-4\cdot10^{-9})}{6} = -6\ V \qquad V_{P2} = k\frac{q_2}{d_2} = \frac{9\cdot10^9\cdot5\cdot10^{-9})}{5} = 9\ V = V_{P3}$

$V_P = V_{P1}+V_{P2}+V_{P3} = -6+9+9 = 12\ V$

$V_{O1} = k\frac{q_1}{d_1} = \frac{9\cdot10^9(-4\cdot10^{-9})}{3} = -12\ V \qquad V_{O2} = k\frac{q_2}{d_2} = \frac{9\cdot10^9\cdot5\cdot10^{-9}}{4} = \frac{45}{4}V = V_{O3}$

$V_O = V_{O1}+V_{O2}+V_{O3} = -12+2\frac{45}{4} = \frac{21}{2}V$

$W = q(V_P-V_O) = 2\cdot10^{-6}(12-\frac{21}{2}) = 3\cdot10^{-6}\ J$ As this work is positive, it's done by the forces of the electric field.

c) If we move the charge q_1 to point (0,y) and the electric potential at point O must be null, then must be verified that:

$$0 = \frac{9\cdot10^9(-4\cdot10^{-9})}{y}+2\frac{9\cdot10^9(5\cdot10^{-9})}{4} \Rightarrow \frac{4}{y} = \frac{5}{2} \Rightarrow y = \frac{8}{5}m$$

So, the point to move q_1 is (0,8/5) m. Obviously, point (0,-8/5) m is also a solution.

2. (2 points) State **Gauss's law** and apply it to calculate the **electric field** created by a **spherical conductor** with radius **R** and charged with charge **Q:** **a)** At a distance **3R** from its centre. **b)** At a distance **R/3** from its centre.	**2. (2 puntos)** Enuncia el **teorema de Gauss** y aplícalo para calcular el **campo eléctrico** creado por un **conductor** esférico de radio **R** cargado con carga **Q**: **a)** A una distancia **3R** de su centro. **b)** A una distancia **R/3** de su centro.

Gauss's law: The net outward flux through any closed surface equals the net charge inside the surface divided by ε_0

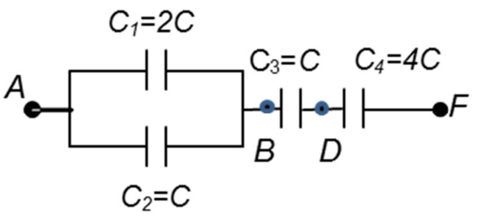
a) Let's consider a spherical surface with radius 3R. At any point of such Surface, electric field and Surface vector are parallel. According Gauss's law, the electric flux trough such surface is

$$\phi = \int_{Sphere} \vec{E} d\vec{S} = \int_{Sphere} EdS = E 4\pi (3R)^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{36\pi\varepsilon_0 R^2}$$

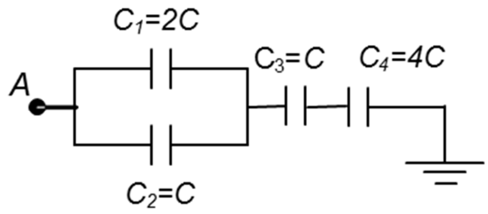
b) Let's consider a spherical surface with radius R/3. According Gauss's law, and taking in account that there isn't any charge inside such surface $\phi = \int_{Sphere} \vec{E} d\vec{S} = \int_{Sphere} EdS = E 4\pi (R/3)^2 = \frac{0}{\varepsilon_0} \Rightarrow E = 0$

3. (2,5 points) The **association of capacitors** on picture is connected to a difference of potential V_A-V_F**=10 V** **a)** Compute the charge on each capacitor (**Q_1, Q_2, Q_3 and Q_4**) and the difference of potential between their terminals (**V_1, V_2, V_3 and V_4**). **b)** The **distance between plates** of capacitor **C_3** is reduced to **a half** and a **dielectric** having relative dielectric permittivity **ε_r=2** is inserted between the plates of capacitor **C_1**. Compute the new **equivalent capacitance** of the set between terminals A and F. **c)** With the conditions given on a), point **F is linked to ground**. Give the electric **potential** of points **B** and **D**.	**3. (2,5 puntos)** La asociación de condensadores de la figura se conecta a una d.d.p. V_A-V_F**=10 V** **a)** Calcula la carga en cada condensador (**Q_1, Q_2, Q_3 y Q_4**) y la diferencia de potencial entre sus terminales (**V_1, V_2, V_3 y V_4**). **b)** La **distancia entre las placas** del condensador **C_3** se reduce a **la mitad** y se introduce un **dieléctrico** de permitividad dieléctrica relativa **ε_r=2** entre las placas del condensador **C_1**. Calcula la nueva **capacidad equivalente** del conjunto entre los terminales A y F. **c)** Manteniendo las condiciones dadas en a), el punto **F se conecta a tierra**. ¿Cuál es el **potencial** eléctrico de los puntos **B** y **D**?.

a)



c)



a) This exercise can be solved in two different ways. Both are correct:

- Without calculating the equivalent capacitance of the association:
  - Capacitors 1 and 2 are connected in parallel and then $V_1 = V_2 \Rightarrow \frac{Q_1}{2C} = \frac{Q_2}{C} \Rightarrow Q_1 = 2Q_2$
  - Capacitors 3 and 4 are connected in series and then $Q_3=Q_4$
  - The charge of capacitor 3 is divided among the plates of capacitors 1 and 2 and then: $Q_1 + Q_2 = Q_3 \Rightarrow Q_1 = \frac{2Q_3}{3}$
  - The difference of potential between A and B equals 10 V: $V_1 + V_3 + V_4 = \frac{Q_1}{2C} + \frac{Q_3}{C} + \frac{Q_4}{4C} = 10$

  From the before equations, we have four equations with four unknowns, and every Q and V can be solved:

  $$\frac{2Q_3}{6C} + \frac{Q_3}{C} + \frac{Q_3}{4C} = 10 \Rightarrow Q_3 = Q_4 = \frac{12}{19} C \cdot 10 = \frac{120}{19} C \Rightarrow Q_1 = \frac{80}{19} C \quad and \quad Q_2 = \frac{40}{19} C$$

  $$V_1 = V_2 = \frac{Q_1}{2C} = \frac{40}{19} V \qquad V_3 = \frac{Q_3}{C} = \frac{120}{19} V \qquad V_4 = \frac{Q_4}{4C} = \frac{30}{19} V$$

  Obviously it is verified that $V_1 + V_3 + V_4 = \frac{40}{19} + \frac{120}{19} + \frac{30}{19} = \frac{190}{19} = 10V$

- By using the equivalent capacitance of the set of capacitors. 1 and 2 are in parallel and then $C_{12} = 2C + C = 3C$. C_{12} is in series with C_3 and C_4. Therefore

$$\frac{1}{C_{eq}}=\frac{1}{C_{12}}+\frac{1}{C_3}+\frac{1}{C_4}=\frac{1}{3C}+\frac{1}{C}+\frac{1}{4C}=\frac{19}{12C}\Rightarrow C_{eq}=\frac{12}{19}C$$

The charge of the equivalent capacitor equals the charge of C_4 and then $Q_4=Q_3=C_{eq}\cdot 10=\frac{12}{19}C\cdot 10=\frac{120}{19}C$

$$V_3=\frac{Q_3}{C}=\frac{120}{19}V \qquad V_4=\frac{Q_4}{4C}=\frac{30}{19}V \qquad V_1=V_2=10-(V_3+V_4)=10-(\frac{120}{19}+\frac{30}{19})=\frac{40}{19}V$$

$$Q_1=2CV_1=\frac{80}{19}C \qquad Q_2=CV_2=\frac{40}{19}C$$

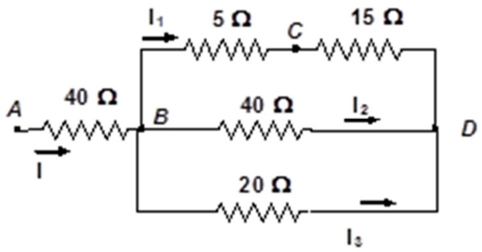
b) With the new conditions, C'_1=4C and C'_3=2C. Therefore, the new equivalent capacitance is:

$$\frac{1}{C'_{eq}}=\frac{1}{5C}+\frac{1}{2C}+\frac{1}{4C}=\frac{19}{20C}\Rightarrow C'_{eq}=\frac{20}{19}C$$

c) If point F is linked to ground, then its potential is zero and potential of point A is 10 V. Therefore

$V_B=V_A-V_1=10-\frac{40}{19}=\frac{150}{19}V$ and $V_D=V_B-V_3=\frac{150}{19}-\frac{120}{19}=\frac{30}{19}V$ Obviously, $V_D=V_4$

4. (2,5 points) Given the association of resistors on picture, it is known that V_{AB} = **10 V**. Find: **a)** **I_1, I_2, I_3, I, V_{CD}, V_{BD}, V_{BC}** and **V_{AD}**. **b)** The equivalent resistance between points **A** and **D.** **c)** The equivalent resistance between points **A** and **C.**	**4. (2,5 puntos)** En la asociación de resistencias de la figura, se sabe que V_{AB} = **10 V**. Calcula: **a)** **I_1, I_2, I_3, I, V_{CD}, V_{BD}, V_{BC}** y **V_{AD}.** **b)** Resistencia equivalente entre **A** y **D.** **c)** Resistencia equivalente entre **A** y **C**.



a) $I=\frac{V_{AB}}{40}=\frac{10}{40}=\frac{1}{4}A$ $\qquad V_{BD}=I_1(5+15)=I_2 40=I_3 20\Rightarrow I_3=2I_2$ and $I_1=2I_2$

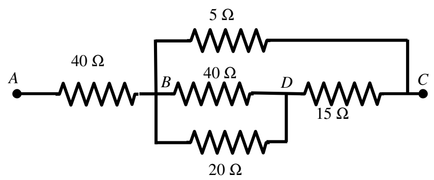
Moreover $I=I_1+I_2+I_3\Rightarrow\frac{1}{4}=2I_2+I_2+2I_2=5I_2\Rightarrow I_2=\frac{1}{20}A \qquad I_1=\frac{1}{10}A \qquad I_3=\frac{1}{10}A$

$$V_{BD}=I_2 40=2V \qquad V_{BC}=I_1 5=\frac{1}{2}V \qquad V_{AD}=V_{AB}+V_{BD}=10+2=12V$$

b) They are three resistors in parallel between B and D: $\frac{1}{R_{BD}}=\frac{1}{20}+\frac{1}{40}+\frac{1}{20}=\frac{5}{40}\Rightarrow R_{BD}=\frac{40}{5}=8\,\Omega$

$$R_{AD}=R_{AB}+R_{BD}=40+8=48\,\Omega$$

c) Between points A and C, the before circuit can be drawn is this way:



Between B and D, resistors of 40 and 20 Ω are connected in parallel: $\frac{1}{R'_{BD}}=\frac{1}{20}+\frac{1}{40}=\frac{3}{40}\Rightarrow R'_{BD}=\frac{40}{3}\Omega$

$$R'_{BC}=\frac{40}{3}+15=\frac{85}{3}\Omega \qquad \frac{1}{R_{BC}}=\frac{1}{\frac{85}{3}}+\frac{1}{5}=\frac{3}{85}+\frac{1}{5}=\frac{20}{85}=\frac{4}{17}\Rightarrow R_{BC}=\frac{17}{4}\Omega \qquad R_{AC}=40+\frac{17}{4}=\frac{177}{4}\Omega$$

Form

Electrostatics

$\vec{F} = \mathrm{K}\frac{\mathrm{q_1 q_2}}{\mathrm{r}^2}\vec{u}_r$ $\vec{E} = \frac{\vec{F}}{\mathrm{q}}$ $\mathrm{K} = \frac{1}{4\pi\varepsilon_0} = 9\cdot 10^9\,\mathrm{(S.I.)}$ $\mathrm{V_A} - \mathrm{V_B} = \int_A^B \vec{E}\cdot\vec{dr}$

$\vec{E} = \mathrm{K}\frac{\mathrm{q}}{\mathrm{r}^2}\vec{u}_r$ $\mathrm{V} = \mathrm{K}\frac{\mathrm{q}}{\mathrm{r}}$ $\int_S \vec{E}\cdot\vec{dS} = \frac{\sum \mathrm{Q}}{\varepsilon_0}$ $W_{AB} = q(V_A - V_B)$

Conductors and capacitors $\mathrm{E} = \frac{\sigma}{\varepsilon_0}$ $\mathrm{C} = \frac{\mathrm{Q}}{\mathrm{V}}$ $\mathrm{C} = \frac{\varepsilon_0 \mathrm{S}}{\mathrm{d}}$

$C_{eq} = \sum C_i$ $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$ $\mathrm{E_d} = \frac{\mathrm{E}}{\varepsilon_\mathrm{r}}$ $C_d = \varepsilon_r C$ $\mathrm{W} = \frac{\mathrm{Q}^2}{2\mathrm{C}} = \frac{\mathrm{QV}}{2} = \frac{\mathrm{V}^2\mathrm{C}}{2}$

Direct Current $\vec{J} = \mathrm{n}\cdot\mathrm{e}\cdot\vec{v}_a$ $\vec{J} = \sigma\cdot\vec{E}$ $\mathrm{R} = \frac{\mathrm{V_1} - \mathrm{V_2}}{\mathrm{I}}$ $\mathrm{R} = \rho\frac{\mathrm{L}}{\mathrm{S}}$

$\rho = \rho_0(1 + \alpha(\mathrm{T} - \mathrm{T_0}))$ $P_R = IV = I^2R = \frac{V^2}{R}$