

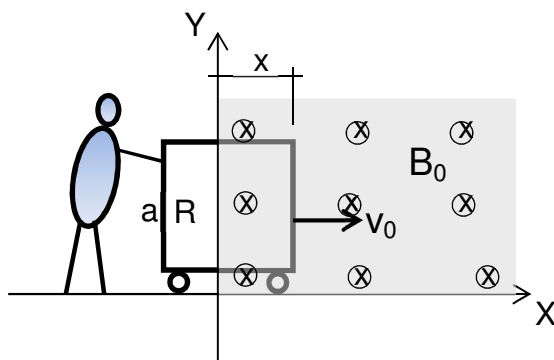


1. (2,5 points) A squared loop with side a and resistance R is placed vertically over a wheeled platform. A worker is pushing such loop (with constant speed v_0) inside an area of the space where a uniform and stationary magnetic field B_0 perpendicular to the loop is acting. When a part of the loop is inside of magnetic field ($0 \leq x \leq a$), compute:

- Magnetic flux ϕ through the loop, as a function of x .
- Induced electromotive force ε on the loop.
- Intensity of current i flowing along the loop, giving its direction.
- Force F done by the worker to move the loop.
- When the loop is completely inside the magnetic field, ¿which is the force done by the worker to move the loop?, reasoning the answer.

1. (2,5 puntos) Una espira cuadrada de lado a y resistencia R se encuentra colocada verticalmente sobre una plataforma con ruedas. Un trabajador introduce dicha espira (con velocidad v_0 constante) en una región del espacio en la que actúa un campo magnético uniforme y estacionario B_0 perpendicular a la espira. Cuando una parte de la espira se encuentra dentro del campo magnético ($0 \leq x \leq a$), calcular:

- Flujo magnético ϕ que atraviesa la espira en función de x .
- Fuerza electromotriz ε inducida en la espira.
- Intensidad de corriente i que circula por la espira, indicando su sentido.
- Fuerza F que debe hacer el trabajador para mover la espira.
- Cuando la espira ha penetrado completamente en el campo magnético, ¿Qué fuerza debe hacer el trabajador para mover la espira? Razonar la respuesta.



Solution

a) As magnetic field is uniform and stationary $\phi = B_0 S = B_0 a x$

$$b) \quad \varepsilon = \frac{d\phi}{dt} = B_0 a \frac{dx}{dt} = B_0 a v_0$$

c) $i = \frac{\varepsilon}{R} = \frac{B_0 a v_0}{R}$ As the flux entering on paper is increasing, the intensity must be counterclockwise

d) The magnetic forces acting on the upper and lower side of the loop cancel each other. Therefore, the resulting magnetic force acting on the loop is that acting on its right side:

$$\vec{F} = i \vec{L} \times \vec{B} = i a \vec{j} \times (-B_0 \vec{k}) = -\frac{B_0^2 a^2 v_0}{R} \vec{i} \quad \text{and the force done by the worker is the opposite: } \vec{F} = \frac{B_0^2 a^2 v_0}{R} \vec{i}$$

e) When the loop is completely inside the magnetic field, magnetic flux through the loop is constant, and no induced current appear. Then, zero force must be done by the worker.

2. (2,5 points) Let's consider a coil having **50 cm** length, **3000 turns** and radius **20 cm**, flowed by an intensity of current **2 A**. A second coil with the same length, **400 turns**, and radius **5 cm** is coaxially placed inside the first one. Compute:

- The magnetic field produced by the first coil at a point of its axis.
- The flux through the second coil produced by the first one.
- The mutual inductance coefficient between both coils.
- If the current along first coil is varying on time according $i(t)=2\cos(100t)$, compute the electromotive force induced on the second coil.

2. (2,5 puntos) Sea un solenoide de **50 cm** de longitud, **3000 espiras**, y **20 cm** de radio, por el que circula una corriente de **2 A**. Un segundo solenoide de la misma longitud, **400 espiras** y **5 cm** de radio está situado coaxialmente dentro del primero. Calcular:

- El campo magnético producido por el primer solenoide en un punto de su eje.
- El flujo que el primer solenoide produce sobre el segundo.
- El coeficiente de inducción mutua entre ambos solenoides.
- Si la corriente en el primer solenoide varía con el tiempo según la expresión $i(t)=2\cos(100t)$, calcula la f.e.m. inducida en el segundo solenoide.

Solution

- By considering the magnetic field uniform inside the coil $B = \mu_0 \frac{3000}{50 \cdot 10^{-2}} 2 = 12\mu_0 10^3 = 48\pi 10^{-4} \text{ T}$
- $\phi = BNS = 12\mu_0 10^3 400\pi (5 \cdot 10^{-2})^2 = 12\mu_0 \pi 10^3 = 48\pi^2 10^{-4} \text{ Wb}$
- $M = \frac{\phi}{I} = \frac{12\mu_0 \pi 10^3}{2} = 6\mu_0 \pi 10^3 = 24\pi^2 10^{-4} \text{ H}$
- $\varepsilon_2 = -\frac{d\phi_2}{dt} = -\frac{d(Mi)}{dt} = -M \frac{di}{dt} = -24\pi^2 10^{-4} (-100 \cdot 2 \cdot \text{sen}(100t)) = 48\pi^2 10^{-2} \text{ sen}(100t) \text{ V}$

3. (2,5 points) A circuit is made up by a **5 Ω** resistor, a **4 mH** inductor and a capacitor with capacitance **100 μF** connected in series (dipole RLC). On terminals of the capacitor is applied a voltage $u_c(t)=10\cos(2000t-100^\circ) \text{ V}$.

- Find the instantaneous magnitudes of intensity, voltage on resistor and inductor, and voltage on terminals of dipole.
- Compute the frequency should have the applied voltage in order the intensity flowing along the dipole was the maximum.

3. (2,5 puntos) Un circuito tiene una resistencia de **5 Ω**, una bobina de **4 mH** y un condensador de **100 μF** conectados en serie. La tensión en los extremos del condensador es $u_c(t)=10\cos(2000t-100^\circ) \text{ V}$.

- Halla la expresión instantánea de la intensidad, la caída de tensión en resistencia y bobina, y la caída de tensión total.
- Calcula la frecuencia que debería tener la tensión aplicada para que la intensidad que circule por el circuito sea máxima.

$$\begin{aligned} \text{a) } X_C &= \frac{1}{C\omega} = \frac{1}{100 \cdot 10^{-6} \cdot 2000} = 5 \Omega & I_m &= \frac{U_m}{X_C} = \frac{10}{5} = 2 \text{ A} & \varphi_i &= -100 + 90 = -10^\circ & i(t) &= 2 \cos(2000t - 10^\circ) \text{ A} \\ X_L &= L\omega = 4 \cdot 10^{-3} \cdot 2000 = 8 \Omega & U_{mL} &= I_m X_L = 2 \cdot 8 = 16 \text{ V} & \varphi_u &= 90 - 10 = 80^\circ & u_L(t) &= 16 \cos(2000t + 80^\circ) \text{ V} \\ U_{mR} &= I_m R = 2 \cdot 5 = 10 \text{ V} & u_R(t) &= 10 \cos(2000t - 10^\circ) \text{ V} \\ Z &= \sqrt{R^2 + (L\omega - 1/C\omega)^2} = \sqrt{5^2 + (8 - 5)^2} = \sqrt{34} \Omega & \text{tg}\varphi &= \frac{X_L - X_C}{R} = \frac{3}{5} = 0,6 \Rightarrow \varphi = 30,96 \approx 31^\circ \\ U_m &= I_m Z = 2 \cdot \sqrt{34} \text{ V} & \varphi_u &= 31 + (-10) = 21^\circ & u(t) &= 2\sqrt{34} \cos(2000t + 21^\circ) \text{ V} \end{aligned}$$

- The intensity will be the maximum when the frequency was the resonant frequency:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} = \frac{1}{2\pi} \sqrt{\frac{1}{4 \cdot 10^{-3} \cdot 100 \cdot 10^{-6}}} = \frac{1580}{2\pi} = 251,5 \text{ Hz}$$

4. (2,5 points) On circuit on picture every diodes have drop forward voltage $V_u=0,7\text{ V}$, and internal resistance negligible (also for the battery).

a) On next table, mark with a cross the correct bias (forward or reverse) for every diode:

Diode	Forward	Reverse
D ₁		
D ₂		
D ₃		

b) Compute intensities I_1 , I_2 and I_3 flowing along diodes D₁, D₂ and D₃.

Compute the differences of potential V_A-V_C and V_C-V_B .

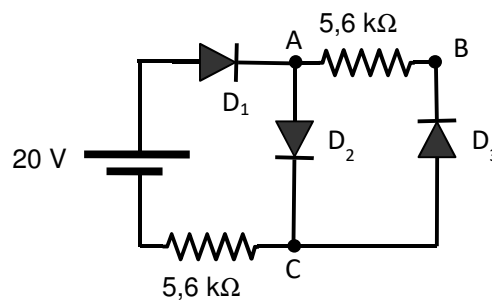
4. (2,5 puntos) En el circuito de la figura, todos los diodos tienen tensión umbral $V_u=0,7\text{ V}$, y resistencia interna despreciable (también para el generador).

a) En la siguiente tabla, marca con una cruz la correcta polarización de cada diodo:

Diodo	Directa	Inversa
D ₁		
D ₂		
D ₃		

b) Calcula las intensidades I_1 , I_2 e I_3 que circulan por los diodos D₁, D₂ y D₃.

Calcula las diferencias de potencial V_A-V_C y V_C-V_B .



a)

Diode	Forward	Reverse
D ₁	X	
D ₂	X	
D ₃		X

b) $I_1 = I_2 = \frac{20 - 0,7 - 0,7}{5,6} = 3,32\text{ mA}$ $I_3 = 0$

c) As D₂ is forward biased $V_A - V_C = 0,7\text{ V}$

On the other hand, as $I_3=0$, $V_A - V_B = 0$ and therefore $V_C - V_B = V_C - V_A = -0,7\text{ V}$

FORM

Magnetic Forces $\vec{F} = q(\vec{v} \times \vec{B})$ $d\vec{F} = Id\vec{\ell} \times \vec{B}$ $\vec{\mu} = N \cdot I \cdot \vec{S}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$ $V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$

Sources of magnetic field $d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \vec{r}}{4\pi r^3}$ $\mu_0 = 4\pi 10^{-7}$ (I.S.units) $B = \frac{\mu_0 I}{2\pi x}$

$B = \frac{\mu_0 I}{2R}$ $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \sum I$ $B = \frac{\mu_0 NI}{l}$

Electromagnetic induction $|\varepsilon| = \frac{d\phi}{dt}$ $\phi = L \cdot I$ $\phi_{21} = M \cdot I_1$ $W_L = \frac{1}{2} L \cdot I^2$

Alternating current $\varphi = \varphi_u - \varphi_i$ $X_L = L\omega$ $X_C = \frac{1}{C\omega}$ $U_{rms} = \frac{U_m}{\sqrt{2}}$ $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$\operatorname{tg}\varphi = \frac{L\omega - 1/C\omega}{R}$$

$$Z = \frac{U_m}{I_m} = \sqrt{R^2 + (L\omega - 1/c\omega)^2}$$

$$P(t) = u(t) \cdot i(t) = U_m I_m \cos\varphi \sin^2 \omega t + \frac{U_m I_m}{2} \sin\varphi \sin \omega t \quad P = U_{\text{rms}} I_{\text{rms}} \cos\varphi \quad f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

Semiconductors

$$n \cdot p = n_i^2$$

$$N_A + n = N_D + p$$