

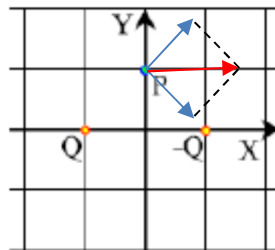


**1. (3 points)** Two point charges  $Q$  and  $-Q$  are placed on vacuum at points  $(-a,0)$  and  $(a,0)$  respectively, as it can be seen on picture.

- Find the total electric field at point  $P(0,a)$ . Draw this total electric field and also the electric field vector created by each charge.
- Compute the electric potential at point  $P$ .
- Compute the work needed to carry a charge  $Q$  from point  $P$  up to the origin of coordinates.
- Is possible to cancel the electric field at point  $P$  by placing a new charge  $Q$  at a point of  $Y$  axis? If yes, find the point where this charge must be placed.

**1. (3 puntos)** Dos cargas puntuales  $Q$  y  $-Q$  se encuentran en el vacío en los puntos  $(-a,0)$  y  $(a,0)$  respectivamente, como se ve en la figura.

- Calcula el campo eléctrico resultante en el punto  $P(0,a)$ . Dibuja los vectores campo eléctrico que crea cada carga y el campo eléctrico resultante.
- Calcula el potencial eléctrico en el punto  $P$ .
- Calcula el trabajo necesario para llevar una carga  $Q$  desde el punto  $P$  hasta el origen de coordenadas.
- ¿Es posible anular el campo eléctrico en  $P$  situando una nueva carga  $Q$  en algún punto del eje  $Y$ ? Si es posible, encuentra el punto donde debe situarse dicha carga.



$$\text{a) } \vec{E}_Q = k \frac{Q \vec{i} + \vec{j}}{r^2 \sqrt{2}} = k \frac{Q \vec{i} + \vec{j}}{2a^2 \sqrt{2}} \quad \vec{E}_{-Q} = k \frac{Q \vec{i} - \vec{j}}{r^2 \sqrt{2}} = k \frac{Q \vec{i} - \vec{j}}{2a^2 \sqrt{2}}$$

$$\vec{E}_P = \vec{E}_Q + \vec{E}_{-Q} = k \frac{Q \vec{i} + \vec{j}}{2a^2 \sqrt{2}} + k \frac{Q \vec{i} - \vec{j}}{2a^2 \sqrt{2}} = k \frac{Q}{\sqrt{2}a^2} \vec{i}$$

$$\text{b) } V_P = V_Q + V_{-Q} = k \frac{Q}{a\sqrt{2}} + k \frac{-Q}{a\sqrt{2}} = 0$$

$$\text{c) } W = Q(V_P - V_0) = Q(0 - 0) = 0$$

- d)** No, it isn't possible. The electric field created by the two charges is horizontal. A new charge placed on  $Y$  axis will always create a vertical electric field at  $P$ , and therefore, it will never be able to cancel the field created by  $Q$  and  $-Q$ .

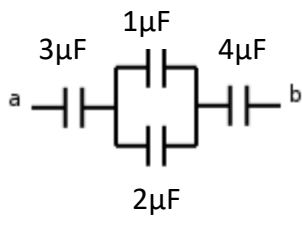
**2. (2 points)** Justify the electric features inside a charged conductor in electrostatic equilibrium (electric field, electric potential, and volumetric density of charge).

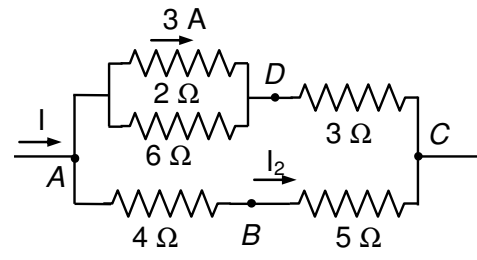
**2. (2 puntos)** Justifica las características eléctricas en el interior de un conductor cargado en equilibrio electrostático (campo eléctrico, potencial eléctrico, y densidad volumétrica de carga).

A conductor is in electrostatic equilibrium when every free electrons are at rest, without motion. This condition can only be reached if the electric field acting on these charges is null, and then, **inside a conductor in electrostatic equilibrium, the electric field is zero.**

If we take two points of conductor, the difference of potential between them can be calculated as the circulation of the electric field along any path joining them (integral of the electric field along this path). If the electric field is zero at every point of this path, then the difference of potential between any two points of conductor is zero. **The electric potential on a conductor in electrostatic equilibrium is constant at any point.**

If we consider any closed surface inside the conductor, the electric flux across such surface is zero (the electric field is zero) and then, by applying Gauss's law, the net charge inside the closed surface is zero. If we consider the surface matching the outer surface of the conductor, then **inside a conductor in electrostatic equilibrium can't exist any charge (volumetric density of charge zero).** The charge of conductor, if exists, must reside on its surface.

<p><b>3. (3 points)</b> The association of capacitors on picture is charged by applying a potential <math>V_{ab} = 11 \text{ V}</math> between points <b>a</b> and <b>b</b>. Compute the <b>equivalent capacitance</b> of the set, the <b>stored energy</b>, and the charge <b>Q</b> and the potential <b>V</b> on terminals of the <b><math>3\mu\text{F}</math></b> and <b><math>2\mu\text{F}</math></b> capacitors.</p>	<p><b>3. (3 puntos)</b> Cargamos la asociación de condensadores de la figura aplicando una <math>V_{ab} = 11 \text{ V}</math> entre los puntos <b>a</b> y <b>b</b>. Calcula la <b>capacidad equivalente</b> del conjunto, la <b>energía almacenada</b> en él, y la carga <b>Q</b> y la diferencia de potencial <b>V</b> de los condensadores de <b><math>3\mu\text{F}</math></b> y <b><math>2\mu\text{F}</math></b>.</p>	
$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{1+2} + \frac{1}{4} = \frac{2}{3} + \frac{1}{4} = \frac{11}{12} \Rightarrow C_{eq} = \frac{12}{11} \mu\text{F}$ $W = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \frac{12}{11} 11^2 = \frac{131}{2} \mu\text{J}$ $Q_3 = Q_{eq} = C_{eq} V = \frac{12}{11} 11 = 12 \mu\text{C} \quad V_3 = \frac{Q_3}{C} = \frac{12}{3} = 4 \text{ V}$ $Q_4 = Q_3 = 12 \mu\text{C} \quad V_4 = \frac{12}{4} = 3 \text{ V} \quad V_{1,2} = V_1 = V_2 = 11 - (V_3 + V_4) = 11 - (4 + 3) = 4 \text{ V} \quad Q_2 = CV = 2 \cdot 4 = 8 \mu\text{C}$		

<p><b>4. (2 puntos)</b> Given the association of resistors on picture, find:  a) <math>V_A - V_D</math>, <math>V_D - V_C</math>, <math>V_A - V_B</math>  b) The equivalent resistance between points <b>A</b> and <b>C</b>.  c) The equivalent resistance between points <b>A</b> and <b>D</b>.</p>	<p><b>4. (2 puntos)</b> Dado el esquema de la figura, calcula:  a) <math>V_A - V_D</math>, <math>V_D - V_C</math>, <math>V_A - V_B</math>  b) Resistencia equivalente entre <b>A</b> y <b>C</b>  c) Resistencia equivalente entre <b>A</b> y <b>D</b>.</p>	
<p>a) <math>V_{AD} = 3 \cdot 2 = 6 \text{ V}</math>      <math>I_{6\Omega} = \frac{V_{AD}}{6} = \frac{6}{6} = 1 \text{ A}</math>      <math>I_{3\Omega} = 3 + 1 = 4 \text{ A}</math>      <math>V_{DC} = 4 \cdot 3 = 12 \text{ V}</math></p> <p><math>V_{AC} = V_{AD} + V_{DC} = 6 + 12 = 18 \text{ V}</math>      <math>I_2 = \frac{V_{AC}}{4+5} = \frac{18}{9} = 2 \text{ A}</math>      <math>V_{AB} = 2 \cdot 4 = 8 \text{ V}</math></p> <p>b) The equivalent resistance of 2 and 6 <math>\Omega</math> in parallel is: <math>\frac{1}{R_{2,6}} = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} \Rightarrow R_{2,6} = \frac{3}{2} \Omega</math></p> <p><math>\frac{1}{R_{AC}} = \frac{1}{\frac{3}{2} + 3} + \frac{1}{4+5} = \frac{1}{\frac{9}{2}} + \frac{1}{9} = \frac{2}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3} \Rightarrow R_{AC} = 3 \Omega</math></p> <p>c) <math>\frac{1}{R_{AD}} = \frac{1}{\frac{3}{2}} + \frac{1}{4+5+3} = \frac{2}{3} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4} \Rightarrow R_{AD} = \frac{4}{3} \Omega</math></p>		

Si te examinas sólo de una parte, debes resolver los cuatro problemas de esa parte.  
*If you are sitting only one part of the exam, then you have to solve the four problems of that part.*

Si te examinas de dos partes, debes resolver los tres primeros problemas de ambas partes.  
*If you are sitting two parts of the exam, then you have to solve the three first problems of both parts.*

Si te examinas de las tres partes, debes resolver los dos primeros problemas de cada una de las partes.  
*If you are sitting the three parts of the exam, then you have to solve the two first problems of every part.*

### Form - Fórmulas

<p>Electrostatics</p>			
$\vec{F} = K \frac{q_1 q_2}{r^2} \vec{u}_r$	$\vec{E} = \frac{\vec{F}}{q}$	$K = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \text{ (S.I.)}$	$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{r}$
$\vec{E} = K \frac{q}{r^2} \vec{u}_r$	$V = K \frac{q}{r}$	$\int_S \vec{E} \cdot d\vec{S} = \frac{\sum Q}{\epsilon_0}$	$W_{AB} = q(V_A - V_B)$

Conductors and capacitors

$$E = \frac{\sigma}{\epsilon_0} \quad C = \frac{Q}{V} \quad C = \frac{\epsilon_0 S}{d}$$

$$C_{eq} = \sum C_i \quad \frac{1}{C_{eq}} = \sum \frac{1}{C_i} \quad E_d = \frac{E}{\epsilon_r} \quad C_d = \epsilon_r C \quad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$$

Direct Current

$$\vec{J} = n \cdot e \cdot \vec{v}_a \quad \vec{J} = \sigma \cdot \vec{E} \quad R = \frac{V_1 - V_2}{I} \quad R = \rho \frac{L}{S}$$

$$\rho = \rho_0(1 + \alpha(T - T_0)) \quad P_R = IV = I^2 R = \frac{V^2}{R}$$