First mid term FFI exam
October, 29th , 2018
Year 2018/19

Applied Physics Dept.

1. (3 points) Given the **two point charges** on picture, **q_1=4 μC** at point **(0,0)** and **q_2=-2 μC** at point **(0,4) m**:

a) Compute the resulting **electric field** vector at point **B (3,0) m**. **Drawn** the electric field created by each charge and the resulting electric field.

b) Compute the electric **potential** at point **B.**

c) Compute the **work** needed to carry a **2 μC** charge **from point B** up to the **infinite**. Is this work done by the forces of the electric field or against them?

d) **Find** a **point** of the **Y axis** where the total **electric field** was **null**. Give its coordinates.

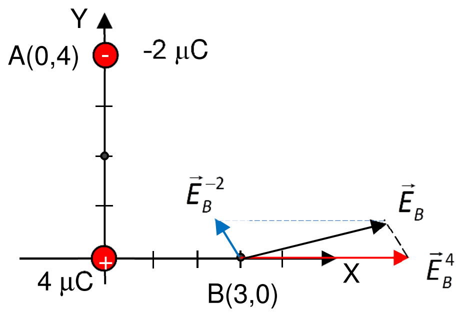
1. (3 puntos) Dadas las **dos cargas puntuales** de la figura, **q_1=4 μC** en el punto **(0,0)** y **q_2=-2 μC** en el punto **(0,4) m**:

a) Calcula el vector **campo eléctrico** resultante en **B (3,0) m. Dibuja** el campo eléctrico creado por cada carga y el campo eléctrico resultante.

b) Calcula el **potencial eléctrico** en **B**.

c) Calcula el **trabajo** necesario para llevar una carga de **2 μC desde el punto B** hasta el infinito. Este trabajo ¿es hecho por las fuerzas del campo, o en contra de ellas?

d) **Encuentra** un **punto** del **eje Y** donde el **campo eléctrico total** se **anule**. Da sus coordenadas.



a) $\vec{E}_B^4 = k\frac{4\cdot 10^{-6}}{3^2}\vec{i} = 9\cdot 10^9\frac{4\cdot 10^{-6}}{3^2}\vec{i} = 4000\vec{i}\ N/C$

$\vec{E}_B^{-2} = k\frac{2\cdot 10^{-6}}{5^2}(\frac{-3\vec{i}+4\vec{j}}{5}) = 9\cdot 10^9\frac{2\cdot 10^{-6}}{5^3}(-3\vec{i}+4\vec{j}) = -432\vec{i}+576\vec{j}\ N/C$

$\vec{E}_B = \vec{E}_B^4 + \vec{E}_B^{-2} = 4000\vec{i} - 432\vec{i} + 576\vec{j} = 3568\vec{i} + 576\vec{j}\ N/C$

b) $V_B = k(\frac{4}{3} - \frac{2}{5})\cdot 10^{-6} = 9\cdot 10^9\frac{14}{15}10^{-6} = 8400\,V$

c) $W = q(V_B - V_\infty) = 2\cdot 10^{-6}(8400-0) = 16,8\cdot 10^{-3}\ J$ As this work is positive, it's done by the forces of the electric field.

d) The electric field can only be null at points over the charge of -2 μC (y>4), because only on this area of Y axis, the electric fields created by both charges can cancel each other. Therefore, If x is the y-coordinate of such point we are looking for, must be verified: $\frac{4}{x^2} = \frac{2}{(x-4)^2}$ This equation shows two solutions $x_1 = 8+4\sqrt{2} = 13,66\ m \quad and \quad x_2 = 8-4\sqrt{2} = 2,34\ m$, corresponding to points (0, 13,66) m and (0, 2,34) m. The last solution is wrong, because on such point both electric fields are reinforced and not cancelled. So, the only correct solution is point (0, 13,66) m.

2. (2 points) The picture shows a hollow and **metallic sphere** with inner and outer radii ***R_1*** and ***R_2***, respectively. A **positive** point **charge**, **Q**, is placed at the centre of the sphere.

a) What is the **surface density of charge** on the inner and outer surfaces?

b) If we **link** the metallic sphere to **ground**, what is the **surface density of charge** on the inner and outer surfaces?

c) After paragraph b), we **disconnect** the metallic sphere from **ground**. What is the **surface density of**

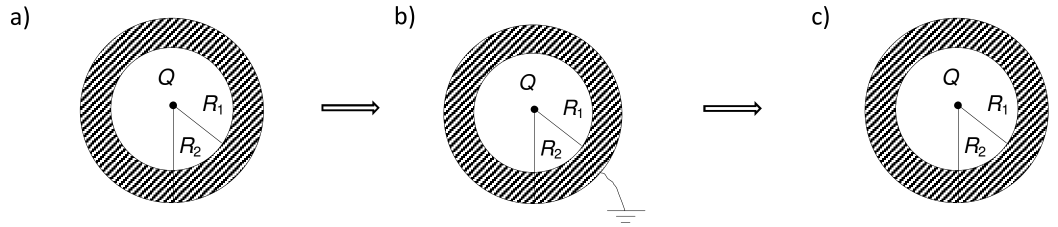
2. (2 puntos) La figura muestra una **esfera metálica** hueca de radios interior y exterior ***R_1*** y ***R_2***, respectivamente. Se coloca una **carga** puntual **positiva**, ***Q***, en el centro de la esfera.

a) ¿Cuál es la **densidad superficial de carga** en las superficies interior y exterior de la esfera?

b) Si **conectamos** la esfera metálica a **tierra**, ¿Cuál es la **densidad superficial de carga** en las superficies interior y exterior de la esfera?

c) Después del punto b), **desconectamos** la esfera metálica de **tierra**. ¿Cuál es la **densidad superficial de**

charge on the inner and outer surfaces? Justify the answers.	**carga** en las superficies interior y exterior de la esfera? Justifica las respuestas.



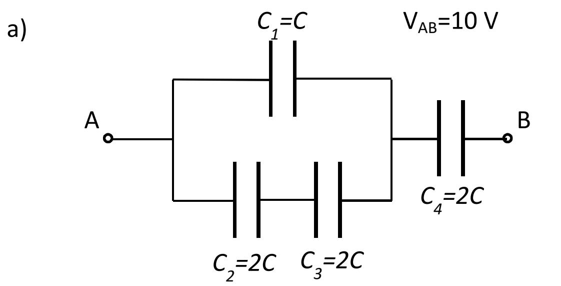
a) As there is total influence between the charge and the inner surface of sphere, the total charge over the inner surface of the sphere must be –Q. And the surface density of charge on this surface: $\sigma_{inner} = \frac{-Q}{4\pi R_1^2}$

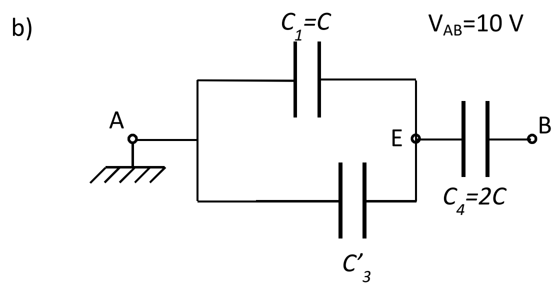
As the sphere is initially discharged, the outer surface of the sphere must take a charge Q, and its density surface of charge will be: $\sigma_{outer} = \frac{Q}{4\pi R_2^2}$

b) If the sphere is linked to ground, the inner surface density of charge remains unchanged: $\sigma_{inner} = \frac{-Q}{4\pi R_1^2}$ and the charge over the outer surface goes to ground in order to cancel the electric field and the potential outside of the sphere. Therefore $\sigma_{outer} = 0$

c) If the ground connection is removed, the total charge of the sphere remains constant. The charge on inner surface of the sphere can't change because of the total influence between point charge and sphere, and then $\sigma_{inner} = \frac{-Q}{4\pi R_1^2}$ and $\sigma_{outer} = 0$

3. (2,5 points) The **association of capacitors** on picture is connected to a difference of potential **V_A-V_B=10 V** **a)** Compute the charge on each capacitor (**Q_1, Q_2, Q_3 and Q_4**) and the difference of potential between their terminals (**V_1, V_2, V_3 and V_4**). **b)** Next, capacitor **2 is removed**, the **distance** between **plates** of capacitor **3 is doubled**, and point **A is grounded**. The power supply is connected at any time, maintaining **10 V between A and B** (figure b). What's the **potential of point E?** And the **charge of capacitor 4?**	**3. (2,5 puntos)** La asociación de condensadores de la figura se conecta a una d.d.p. **V_A-V_B=10 V** **a)** Calcula la carga en cada condensador (**Q_1, Q_2, Q_3 y Q_4**) y la diferencia de potencial entre sus terminales (**V_1, V_2, V_3 y V_4**). **b)** A continuación, el condensador **2 es eliminado**, se **dobla la separación** entre las **placas** del condensador **3**, y el punto **A se conecta a tierra**. La fuente de tensión está siempre conectada, manteniendo **10 V entre A y B** (figura b). Cuál es el potencial del punto E? Y la **carga del condensador 4**?





a) This exercise can be solved in two different ways. Both are correct:

- Without calculating the equivalent capacitance of the association:

- Capacitors 2 and 3 are connected in series and then $Q_2=Q_3=Q_{23}$.
- Capacitors 2 and 3 are in parallel with 1 and then $V_1 = V_2 + V_3 \Rightarrow \frac{Q_1}{C} = \frac{Q_{23}}{2C} + \frac{Q_{23}}{2C} \Rightarrow Q_{23} = Q_1$
- The charge taken by 1 and 3 equals the charge on 4: $Q_1 + Q_3 = Q_4 \Rightarrow Q_4 = 2Q_1$
- Moreover, the difference of potential of this association is 10 V:

$$V_1 + V_4 = \frac{Q_1}{C} + \frac{Q_4}{2C} = 10 \Rightarrow \frac{Q_1}{C} + \frac{2Q_1}{2C} = 10 \Rightarrow \frac{4Q_1}{2C} = 10 \Rightarrow Q_1 = 5C = Q_2 = Q_3 \qquad Q_4 = 10C$$

- And the differences of potential: $V_1 = \frac{Q_1}{C} = 5\,V \qquad V_2 = \frac{Q_2}{2C} = \frac{5}{2}V = V_3 \qquad V_4 = \frac{Q_4}{2C} = 5\,V$

Obviously it is verified that $V_2+V_3+V_4=\frac{5}{2}+\frac{5}{2}+5=10\,V$

- By using the equivalent capacitance of the set of capacitors.

2 and 3 are in series and then $\frac{1}{C_{23}}=\frac{1}{2C}+\frac{1}{2C}=\frac{2}{2C}=\frac{1}{C}\Rightarrow C_{23}=C$

2 and 3 are in parallel with 1 and then $C_{123}=C_1+C_{23}=C+C=2C$.

C_{123} is in series with C_4. Therefore $\frac{1}{C_{eq}}=\frac{1}{2C}+\frac{1}{2C}=\frac{1}{C}\Rightarrow C_{eq}=C$

The charge of the equivalent capacitor equals the charge of C_4 and then $Q_4=C_{eq}\cdot 10=10C$

$V_4=\frac{Q_4}{2C}=\frac{10C}{2C}=5\,V \qquad V_1=V_{23}=10-V_4=10-5=5\,V\Rightarrow Q_1=CV_1=5C$

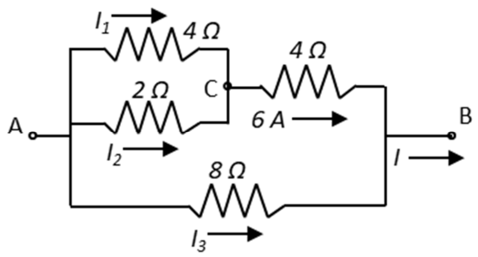
$V_{23}=\frac{Q_{23}}{2C}+\frac{Q_{23}}{2C}=\frac{Q_{23}}{C}=5\,V\Rightarrow Q_{23}=Q_2=Q_3=5C$ And $V_2=\frac{Q_2}{2C}=\frac{5C}{2C}=\frac{5}{2}\,V=V_3$

b) With the new conditions, C'_3=C and the equivalent capacitance of 1 and 3: C'_{13}=2C. Note that C'_{13} equals the equivalent capacitance C_{123} of before paragraph. Then, the situation for C_4 is the same than before, and both its charge and its potential remain equal: $V_4=5\,V$ $Q_4=10C$ As V_A=0 and V_A-V_B=10 therefore V_B=-10 V
V_E-V_B=5→V_E=V_B+5=-5 V

Anyway, if you don't note that the situation remains unchanged for C_4, you can still solve the exercise:
C'_{13}=2C. This equivalent capacitor equals C_4 and they are in series, being the potential equally divided between them: $V_1=V_3=5\,V$ and $V_4=5\,V$

As V_A=0 and V_A-V_B=10 therefore V_B=-10 V V_E-V_B=5→V_E=V_B+5=-5 V and $Q_4=2C\cdot 5=10C$

4. (2,5 points) Given the **association** of resistors on picture, and **the shown intensity**, find: **a)** **I_1, I_2, I_3, I, V_{AC}, V_{CB}** and **V_{AB}.** **b)** If point **C is grounded**, give **V_A** and **V_B**. **c)** The equivalent resistance between points **A** and **C.**	**4. (2,5 puntos)** Dada la **asociación** de resistencias de la figura, y la **intensidad mostrada**, calcula: **a)** **I_1, I_2, I_3, I, V_{AC}, V_{CB}** y **V_{AB}.** **b)** Si el punto **C se conecta a tierra**, halla **V_A** and **V_B**. **c)** Resistencia equivalente entre los puntos **A** y **C**.

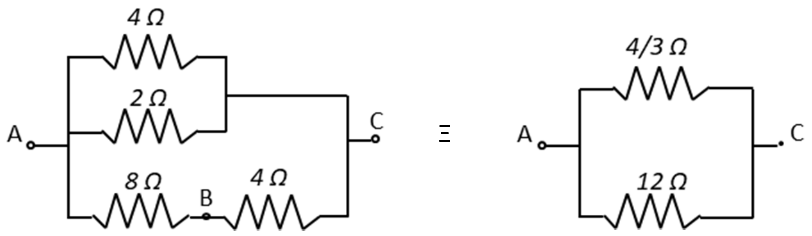


a) 4 Ω and 2 Ω are connected in parallel. Therefore $\left.\begin{array}{l}I_1+I_2=6\\4I_1=2I_2\end{array}\right\}\Rightarrow I_1=2\,A \quad I_2=4\,A \qquad V_{AC}=4I_1=4\cdot 2=8\,V$

$V_{CB}=4\cdot 6=24\,V \qquad V_{AB}=V_{AC}+V_{CB}=8+24=32\,V \qquad I_3=\frac{V_{AB}}{8}=\frac{32}{8}=4\,A \qquad I=I_3+6=4+6=10\,A$

b) If C is grounded, V_C=0. By taking in account the differences of potential computed on before paragraph:
$V_{AC}=V_A-V_C=8\Rightarrow V_A=8+V_C=8\,V \qquad V_{CB}=V_C-V_B=24\Rightarrow V_B=V_C-24=-24\,V$

c) Between points A and C, the before circuit can be drawn is this way:



And the equivalent resistance between A and C: $\frac{1}{R_{AC}}=\frac{3}{4}+\frac{1}{12}=\frac{10}{12}\Rightarrow R_{AC}=\frac{12}{10}=\frac{6}{5}\,\Omega$

Form

Electrostatics

$\vec{F}=K\frac{q_1q_2}{r^2}\vec{u}_r$ $\quad \vec{E}=\frac{\vec{F}}{q}$ $\quad K=\frac{1}{4\pi\varepsilon_0}=9\cdot10^9\,(S.I.)$ $\quad \varepsilon_0=8{,}85\cdot10^{-12}\,(S.I.)$

$V_A-V_B=\int_A^B\vec{E}\cdot\vec{dr}$ $\quad \vec{E}=K\frac{q}{r^2}\vec{u}_r$ $\quad V=K\frac{q}{r}$ $\quad \int_S\vec{E}\cdot\vec{dS}=\frac{\sum Q}{\varepsilon_0}$ $\quad W_{AB}=q(V_A\text{-}V_B)$

Conductors and capacitors $\quad E=\frac{\sigma}{\varepsilon_0}$ $\quad C=\frac{Q}{V}$ $\quad C=\frac{\varepsilon_0 S}{d}$ $\quad C_{eq}=\sum C_i$

$\frac{1}{C_{eq}}=\sum\frac{1}{C_i}$ $\quad E_d=\frac{E}{\varepsilon_r}$ $\quad C_d=\varepsilon_r C$ $\quad W=\frac{Q^2}{2C}=\frac{QV}{2}=\frac{V^2C}{2}$

Direct Current $\quad \vec{J}=n\cdot e\cdot\vec{v}_d$ $\quad \vec{J}=\sigma\vec{E}$ $\quad R=\frac{V_1-V_2}{I}$ $\quad R=\rho\frac{L}{S}$

$\rho=\rho_0(1+\alpha(T-T_0))$ $\quad P_R=IV=I^2R=\frac{V^2}{R}$