



1. (2,5 points) Two point charges with $-2 \mu\text{C}$ and $4 \mu\text{C}$ are placed on vacuum at points **A (8,0) m** and **B (-4,0) m** respectively, as it can be seen on picture. Find:

- The total electric field at point **C (2,-8) m**. Draw the electric field vector created by each charge and the resulting electric field.
- The work** done by the electric field to carry a $-3 \mu\text{C}$ charge from point **C** up to the **infinite**. Who is doing this work?
- A point **P** of X axis where the total **electric field** was null.

1. (2,5 puntos) Dos cargas puntuales de $-2 \mu\text{C}$ y $4 \mu\text{C}$ se encuentran en el vacío en los puntos **A (8,0) m** y **B (-4,0) m** respectivamente. Calcula:

- El campo eléctrico resultante en el punto **C (2,-8) m**. **Dibuja** los vectores campo eléctrico que crea cada carga y el campo eléctrico resultante.
- El trabajo** realizado por el campo eléctrico al desplazar una carga de $-3 \mu\text{C}$ desde el punto **C** al **infinito**. ¿Quién realiza el trabajo?
- Encuentra un **punto P** del eje X donde el **campo** eléctrico total sea **cero**.

Solution:

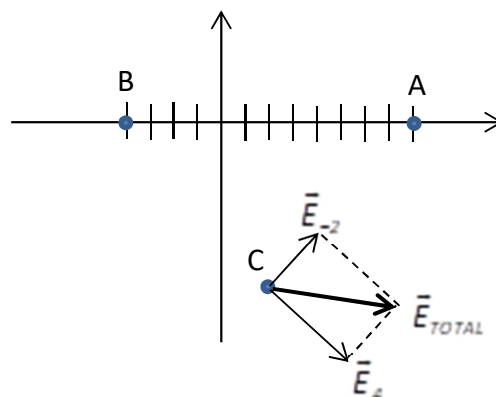
$$a) \vec{AC} = (2, -8) - (8, 0) = (-6, -8) \Rightarrow |\vec{AC}| = \sqrt{6^2 + 8^2} = 10 \text{ m}$$

$$\vec{u}_{AC} = \frac{1}{10}(-6, -8) \vec{E}_{TOTAL}$$

$$\vec{BC} = (2, -8) - (-4, 0) = (6, -8) \Rightarrow |\vec{BC}| = \sqrt{6^2 + 8^2} = 10 \text{ m}$$

$$\vec{u}_{BC} = \frac{1}{10}(6, -8)$$

$$\vec{E}_C = k \frac{-2 \cdot 10^{-6}}{10^2} \frac{(-6, -8)}{10} + k \frac{4 \cdot 10^{-6}}{10^2} \frac{(6, -8)}{10} = k \frac{2 \cdot 10^{-6}}{10^3} ((6, 8) + 2(6, -8)) = 18(18, -8) = 324\vec{i} - 144\vec{j} \text{ N/C}$$



$$b) V_C = k \frac{-2 \cdot 10^{-6}}{10} + k \frac{4 \cdot 10^{-6}}{10} = k \frac{2 \cdot 10^{-6}}{10} = 18 \cdot 10^2 = 1800 \text{ V}$$

$$V_\infty = 0$$

$$W_{C\infty} = Q(V_C - V_\infty) = -3 \cdot 10^{-6} (1800 - 0) = -54 \cdot 10^{-4} \text{ J}$$

As the work is negative, it means that it is done by external forces against the electric field.

- Such point P can only be found on right of point A. If the coordinates of such point P are (X,0), then must be verified that:

$$k \frac{4 \cdot 10^{-6}}{(x+4)^2} = k \frac{2 \cdot 10^{-6}}{(x-8)^2} \Rightarrow 2(x-8)^2 = (x+4)^2 \Rightarrow x_1 \approx 37 \text{ m} \text{ and } x_2 \approx 3 \text{ m}$$

The solution of X_2 corresponds to a point between A and B, where the electric can't be null. Therefore:

$$P(37,0) \text{ m}$$

2. (2,5 points)

a) **State Gauss's law.**

Apply it to calculate the electric field created by an **homogeneous** spherical distribution of charge with radius **2 m** and density of charge **4 C/m³** at a distance

- b) **R/3** of its centre.
c) **3R** of its centre.

2. (2,5 puntos)

a) **Enuncia el teorema de Gauss.**

Aplicalo para calcular el campo eléctrico creado por una esfera de radio **2 m** cargada con una densidad volumétrica de carga **homogénea de 4 C/m³** a una distancia

- b) **R/3** de su centro.
c) **3R** de su centro.

Solution:

a) Gauss's law: The flux of the electric field across a closed surface equals the charge enclosed inside the closed divided into ϵ_0 .

b) The electric field created by the homogeneous charge enclosed inside a sphere will have radial direction. If we consider a spherical surface with the same centre than the distribution of charge and radius $R/3 < R$, then the charge enclosed inside this sphere is

$$Q_{enc} = 4 \cdot \frac{4}{3} \pi \left(\frac{R}{3}\right)^3 = \frac{16}{81} \pi R^3$$

And the flux across the considered surface: $\phi = \int_{\text{sphere } R/3} \vec{E} d\vec{S} = E \cdot 4\pi \left(\frac{R}{3}\right)^2$

Application of Gauss's law leads to: $\phi = E \cdot 4\pi \left(\frac{R}{3}\right)^2 = \frac{16}{81} \pi R^3 \Rightarrow E(R/3) = \frac{4R}{9\epsilon_0} = \frac{8}{9\epsilon_0} = 32\pi 10^9 \text{ N/C}$

c) In the same way than on before case, but taking a sphere with radius 3R:

$$Q_{enc} = 4 \cdot \frac{4}{3} \pi R^3 = \frac{16}{3} \pi R^3 \quad \text{and} \quad \phi = E \cdot 4\pi (3R)^2 = \frac{16}{3} \pi R^3 \Rightarrow E(3R) = \frac{16\pi R^3}{3 \cdot 4\pi \cdot 9R^2 \epsilon_0} = \frac{4R}{27\epsilon_0} = \frac{8}{27\epsilon_0} = \frac{32}{3} \pi 10^9 \text{ N/C}$$

3. (2,5 points) Two capacitors (capacitances **C** (1) y **2C** (2)) are connected to a source giving a difference of potential **V₀** between its terminals.

a) Find the charge on each capacitor, **Q₁** and **Q₂**.

The source is removed, and a dielectric with dielectric relative permittivity $\epsilon_r=2$ is inserted between the plates of capacitor (2). Compute:

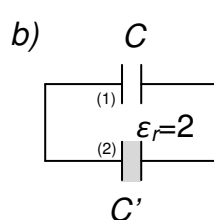
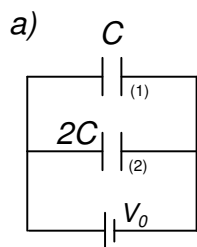
- b) The new capacitance of capacitor (2), **C'**.
c) The charge on each capacitor, **Q'₁** and **Q'₂**.
d) The difference of potential between the plates of both capacitors, **V'**.
e) The stored energy on capacitor (2), **W₂**.

3. (2,5 puntos) Dos condensadores (capacidades **C** (1) y **2C** (2)) se conectan a una fuente de tensión que da una diferencia de potencial **V₀** entre sus terminales.

a) Calcula la carga en cada condensador, **Q₁** y **Q₂**.

Se retira la fuente y después se inserta un dieléctrico de permitividad dieléctrica relativa $\epsilon_r=2$ entre las placas del condensador (2). Calcula:

- b) La nueva capacidad del condensador (2), **C'**.
c) La carga en cada condensador, **Q'₁** y **Q'₂**.
d) La diferencia de potencial entre las placas de ambos condensadores, **V'**.
e) La energía almacenada en el condensador (2), **W₂**.



Solution:

a) $Q_1 = CV_0$ $Q_2 = 2CV_0$

b) $C' = 2 \cdot 2C = 4C$

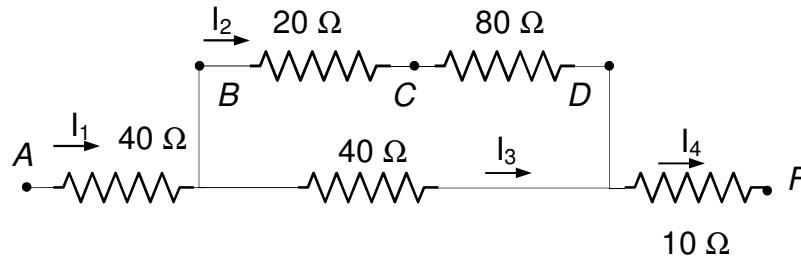
c) $Q'_1 + Q'_2 = Q_1 + Q_2 = 3CV_0$ and $\frac{Q'_1}{C} = \frac{Q'_2}{4C} \Rightarrow Q'_2 = 4Q'_1$ From these equations: $Q'_1 = \frac{3}{5} CV_0$ and $Q'_2 = \frac{12}{5} CV_0$

d) $V' = \frac{Q'_1}{C} = \frac{Q'_2}{4C} = \frac{3}{5} V_0$

e) $W_2 = \frac{1}{2} C' V'^2 = \frac{1}{2} 4C \left(\frac{3}{5} V_0\right)^2 = \frac{18}{25} CV_0^2$

- 4. (2,5 points)** Given the association of resistors on picture, is known that $V_{AF}=V_A-V_F=10\text{ V}$. Find:
- The equivalent resistance between points **B** and **D**.
 - The equivalent resistance between points **A** and **F**.
 - $I_1, I_2, I_3, I_4, V_{CD}, V_{BD}, V_{AB}$ and V_{DF}

- 4. (2,5 puntos)** Dada la asociación de resistencias de la figura, se sabe que $V_{AF}=V_A-V_F=10\text{ V}$. Calcula:
- Resistencia equivalente entre **B** y **D**.
 - Resistencia equivalente entre **A** y **F**.
 - $I_1, I_2, I_3, I_4, V_{CD}, V_{BD}, V_{AB}$ y V_{DF}



$$a) \frac{1}{R_{BD}} = \frac{1}{20+80} + \frac{1}{40} = \frac{7}{200} \Rightarrow R_{BD} = \frac{200}{7} \Omega$$

$$b) R_{AF} = 40 + \frac{200}{7} + 10 = \frac{550}{7} \Omega$$

$$c) I_1 = I_4 = \frac{10}{\frac{550}{7}} = \frac{7}{55} \text{ A} \quad V_{AB} = \frac{7}{55} \cdot 40 = \frac{56}{11} \text{ V} \quad V_{DF} = \frac{7}{55} \cdot 10 = \frac{14}{11} \text{ V} \quad V_{BD} = V_{AF} - V_{AB} - V_{DF} = 10 - \frac{56}{11} - \frac{14}{11} = \frac{40}{11} \text{ V}$$

$$I_3 = \frac{40}{\frac{11}{40}} = \frac{1}{11} \text{ A} \quad I_2 = \frac{40}{\frac{11}{100}} = \frac{2}{55} \text{ A} \quad \text{and} \quad V_{CD} = \frac{2}{55} \cdot 80 = \frac{32}{11} \text{ V}$$

Si te examinas sólo de una parte, debes resolver los cuatro problemas de esa parte.

If you are sitting only one part of the exam, then you have to solve the four problems of that part.

Si te examinas de dos partes, debes resolver los tres primeros problemas de ambas partes.

If you are sitting two parts of the exam, then you have to solve the three first problems of both parts.

Si te examinas de las tres partes, debes resolver los dos primeros problemas de cada una de las partes.

If you are sitting the three parts of the exam, then you have to solve the two first problems of every part.

Form - Fórmulas

Electrostatics

$$\vec{F} = k \frac{q_1 q_2}{r^2} \vec{u}_r \quad \vec{E} = \frac{\vec{F}}{q} \quad K = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \text{ (S.I.)} \quad V_A - V_B = \int_A^B \vec{E} \cdot d\vec{r}$$

$$\vec{E} = K \frac{q}{r^2} \vec{u}_r \quad V = K \frac{q}{r} \quad \int_S \vec{E} \cdot d\vec{S} = \frac{\sum Q}{\epsilon_0} \quad W_{AB} = q(V_A - V_B)$$

$$\text{Conductors and capacitors} \quad E = \frac{\sigma}{\epsilon_0} \quad C = \frac{Q}{V} \quad C = \frac{\epsilon_0 S}{d}$$

$$C_{eq} = \sum C_i \quad \frac{1}{C_{eq}} = \sum \frac{1}{C_i} \quad E_d = \frac{E}{\epsilon_r} \quad C_d = \epsilon_r C \quad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$$

$$\text{Direct Current} \quad \vec{J} = n \cdot e \cdot \vec{v}_d \quad \vec{J} = \sigma \vec{E} \quad R = \frac{V_1 - V_2}{I} \quad R = \rho \frac{L}{S}$$

$$\rho = \rho_0 (1 + \alpha(T - T_0)) \quad P_R = IV = I^2 R = \frac{V^2}{R}$$