

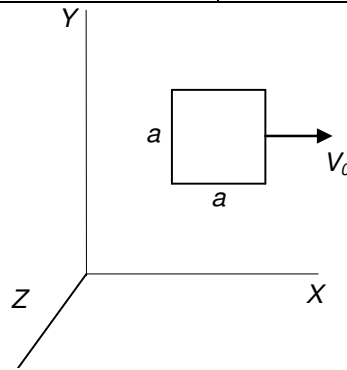


1. (2,5 points) A squared loop with side a and resistance R is moving (the whole loop is moving) inside a **uniform** and **not stationary** magnetic field $\vec{B} = 3t^2\vec{k} \text{ T}$ (t is the time in seconds) with constant velocity $\vec{v} = v_0\vec{j}$. Find:

- Magnetic flux ϕ through the loop, as a function of t .
- Induced electromotive force \mathcal{E} on the loop.
- Intensity of current i flowing along the loop, giving its direction.
- Force \vec{F} (in vector form) acting on the left side of the loop when $t=1 \text{ s}$.
- Compute the intensity of current would flow along the loop and its direction if the whole loop is moving upwards with velocity $\vec{v} = v_0\vec{j}$.

1. (2,5 puntos) Una espira cuadrada de lado a y resistencia R se mueve (la espira completa) dentro de un campo magnético **uniforme** y **no estacionario** $\vec{B} = 3t^2\vec{k} \text{ T}$ (t es el tiempo en segundos). Calcula:

- Flujo magnético ϕ que atraviesa la espira en función de t .
- Fuerza electromotriz \mathcal{E} inducida en la espira.
- Intensidad de corriente i que circula por la espira, indicando su sentido.
- Fuerza \vec{F} (en forma vectorial) que actúa sobre el lado izquierdo de la espira en el instante $t=1 \text{ s}$.
- Calcula la intensidad de corriente que circularía por la espira y su sentido si la espira completa se moviera hacia arriba con velocidad $\vec{v} = v_0\vec{j}$.



Solution

a) As magnetic field is uniform and parallel to surface vector: $\phi = \int_S \vec{B} \cdot d\vec{S} = \int_S B \cdot dS = B \int_S dS = BS = 3t^2 a^2$

b) $\mathcal{E} = \frac{d\phi}{dt} = 6ta^2$

c) $i = \frac{\mathcal{E}}{R} = \frac{6ta^2}{R}$ The magnetic field is exiting from the paper, and then the exiting flux is increasing on time.

Therefore, the induced current must create a magnetic field (and a flux) entering on paper. To do it, the induced current must be clockwise.

d) $\vec{B}(t=1) = 3 \cdot 1^2 \vec{k} = 3\vec{k} \text{ T}$ $i(t=1) = \frac{6 \cdot 1 \cdot a^2}{R} = \frac{6a^2}{R}$

As magnetic field is uniform: $\vec{F}(t=1) = i(t=1)(\vec{L} \times \vec{B}(t=1)) = \frac{6a^2}{R} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & a & 0 \\ 0 & 0 & 3 \end{vmatrix} = \frac{6a^2}{R} 3a\vec{i} = \frac{18a^3}{R} \vec{i}$

- e) If the loop is moving upwards, the flux, the induced current and its direction are equal than those of a) b) and c). As the magnetic field only depends on time, it doesn't matter the velocity of the loop.

2. (2,5 points) Let's consider a coil having **80 turns**, radius **5 cm**, and a resistance of **30 Ω**. A uniform magnetic field increasing on time $B = kt$ (k is a positive constant, and t is the time in seconds) is acting parallel to the axis of the coil. Compute:

- The magnetic flux on the coil.
- The electromotive force induced on the coil.
- The induced current on the coil.
- Compute the value of constant k in order the induced current on the coil was a **4 A** constant current.
- If the induced current on the coil is 4 A, compute the voltage on terminals of coil.

2. (2,5 puntos) Sea un solenoide **80 espiras**, **5 cm** de radio, y una resistencia de **30 Ω**. Un campo magnético uniforme creciente en el tiempo $B = kt$ (k es una constante positiva, y t es el tiempo en segundos) actúa paralelamente al eje del solenoide. Calcula:

- El flujo magnético en el solenoide.
- La fuerza electromotriz inducida en el solenoide.
- La corriente inducida en el solenoide.
- Calcula el valor de la constante k para que la corriente inducida en el solenoide sea una corriente constante de **4 A**.
- Si la corriente inducida en el solenoide es 4 A, calcula la tensión en los terminales del solenoide.

Solution

a) By considering the magnetic field uniform inside the coil:

$$\phi = BNS = kt \cdot 80 \cdot \pi (5 \cdot 10^{-2})^2 = kt\pi \cdot 80 \cdot 25 \cdot 10^{-4} = 2kt\pi \cdot 10^{-1}$$

$$b) \quad \varepsilon = \frac{d\phi}{dt} = 2k\pi 10^{-1}$$

$$c) \quad i = \frac{\varepsilon}{R} = \frac{2k\pi 10^{-1}}{30} = \frac{k\pi 10^{-1}}{15}$$

$$d) \quad i = \frac{k\pi 10^{-1}}{15} = 4 \Rightarrow k = \frac{600}{\pi} = 191 \text{ T/s}$$

e) As the current is constant, there is no voltage on terminals of the coil because of its self-inductance. The only voltage is due to the resistance of the coil. Therefore: $V_{coil} = iR = 4 \cdot 30 = 120 \text{ V}$

3. (2,5 points) Two unknown basic dipoles are connected in series, being $i(t) = 15\cos(500t + 30^\circ)$ the intensity flowing along them, and $u(t) = 60\sqrt{2}\cos(500t - 15^\circ)$ the difference of potential between the terminals of the association. Find:

- Which two elements are associated in series?
- Compute the magnitude of these elements associated in series.
- Compute the instantaneous voltage (u(t)) on each element.

3. (2,5 puntos) Dos dipolos básicos desconocidos están asociados en serie, siendo $i(t) = 15\cos(500t + 30^\circ)$ la corriente que circula por ellos, y $u(t) = 60\sqrt{2}\cos(500t - 15^\circ)$ la diferencia de potencial entre los terminales de la asociación. Halla:

- Cuáles son los dos elementos asociados en serie?
- Calcula los valores de ambos elementos asociados en serie.
- Calcula la tensión instantánea en cada uno de los elementos.

Solution:

a) $\varphi = -15 - 30 = -45^\circ$ As phase lag is different than 0° , -90° or 90° , and it is negative, then a resistor and a capacitor must be those elements connected in series.

$$b) \quad \operatorname{tg}\varphi = \operatorname{tg}(-45^\circ) = -1 = \frac{-X_c}{R} \Rightarrow X_c = R$$

$$\text{Moreover,} \quad Z = \frac{U_m}{I_m} = \frac{60\sqrt{2}}{15} = 4\sqrt{2}$$

$$R = Z \cos \varphi = 4\sqrt{2} \cos(-45^\circ) = 4\sqrt{2} \frac{\sqrt{2}}{2} = 4 \Omega$$

$$X_c = 4\sqrt{2} \sin(45^\circ) = 4\sqrt{2} \frac{\sqrt{2}}{2} = 4 \Omega \Rightarrow C = \frac{1}{X_c \omega} = \frac{1}{4 \cdot 500} = 500 \mu\text{F}$$

$$c) \quad u_R(t) = 4 \cdot 15 \cos(500t + 30^\circ) = 60 \cos(500t + 30^\circ) \text{ V}$$

$$u_C(t) = 4 \cdot 15 \cos(500t - 60^\circ) = 60 \cos(500t - 60^\circ) \text{ V}$$

<p>4. (2,5 points) An extrinsic n type semiconductor is made up by silicon doped with donor atoms. The concentration of free electrons in the semiconductor is $n=3 \cdot 10^{19} \text{ e}^-/\text{cm}^3$ and the intrinsic carrier density of silicon at 300 K is $n_i=1,5 \cdot 10^{16} \text{ cm}^{-3}$.</p> <p>a) Compute the concentration of holes in the semiconductor at 300 K.</p> <p>b) ¿What is the concentration of donor atoms in the semiconductor?</p> <p>c) ¿Which would be the density of free electrons (n) and holes (p) at 300 K if the semiconductor wasn't doped?</p>	<p>4. (2,5 puntos) Un semiconductor extrínseco tipo n está hecho de Si dopado con átomos donadores. La concentración de electrones libres en el semiconductor es $n=3 \cdot 10^{19} \text{ e}^-/\text{cm}^3$ y la concentración intrínseca de portadores del Si a 300 K es $n_i=1,5 \cdot 10^{16} \text{ cm}^{-3}$.</p> <p>a) Calcula la concentración de huecos en el semiconductor a 300 K.</p> <p>b) ¿Cuál es concentración de átomos donadores en el semiconductor?</p> <p>c) ¿Cuál sería la concentración de electrones libres y huecos a 300 K si el semiconductor no estuviera dopado?</p>
<p>Solution:</p> <p>a) If we assume that the concentration of donor atoms is very high compared to the intrinsic carrier density, then $n=N_D$ and $N_D=3 \cdot 10^{19} \text{ donor atoms/cm}^3$. The ratio of donor atoms to the intrinsic carrier density is higher than 10^3 and then our assumption is correct (error less than 0,1%).</p> <p>b) $p = \frac{n_i^2}{n} = \frac{(1,5 \cdot 10^{16})^2}{3 \cdot 10^{19}} = 0,75 \cdot 10^{13} \text{ holes/cm}^3$</p> <p>c) If semiconductor wasn't doped, the density of free electrons and holes at 300 K would be the intrinsic carrier density: $n = p = 1,5 \cdot 10^{16} \text{ carrier/cm}^3$</p>	

Si te examinas sólo de una parte, debes resolver los cuatro problemas de esa parte.

If you are sitting only one part of the exam, then you have to solve the four problems of that part.

Si te examinas de dos partes, debes resolver los tres primeros problemas de ambas partes.

If you are sitting two parts of the exam, then you have to solve the three first problems of both parts.

Si te examinas de las tres partes, debes resolver los dos primeros problemas de cada una de las partes.

If you are sitting the three parts of the exam, then you have to solve the two first problems of every part.

FORM

Magnetic Forces $\vec{F} = q(\vec{v} \times \vec{B})$ $d\vec{F} = Id\vec{l} \times \vec{B}$ $\vec{\mu} = N \cdot I \cdot \vec{S}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$ $V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$

Sources of magnetic field $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$ $\mu_0 = 4\pi 10^{-7} \text{ (I.S.units)}$ $B = \frac{\mu_0 I}{2\pi x}$

$B = \frac{\mu_0 I}{2R}$ $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \sum I$ $B = \frac{\mu_0 NI}{l}$

Electromagnetic induction $|\varepsilon| = \frac{d\phi}{dt}$ $\phi = L \cdot I$ $\phi_{21} = M \cdot I_1$ $W_L = \frac{1}{2} L \cdot I^2$

Alternating current $\varphi = \varphi_u - \varphi_i$ $X_L = L\omega$ $X_C = \frac{1}{C\omega}$ $U_{rms} = \frac{U_m}{\sqrt{2}}$ $I_{rms} = \frac{I_m}{\sqrt{2}}$

$tg\varphi = \frac{L\omega - 1/C\omega}{R}$ $Z = \frac{U_m}{I_m} = \sqrt{R^2 + (L\omega - 1/c\omega)^2}$

$P(t) = u(t) \cdot i(t) = U_m I_m \cos\varphi \sin^2 \omega t + \frac{U_m I_m}{2} \sin\varphi \sin 2\omega t$ $P = U_{rms} I_{rms} \cos\varphi$ $f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$

Semiconductors $n \cdot p = n_i^2$ $N_A + n = N_D + p$