

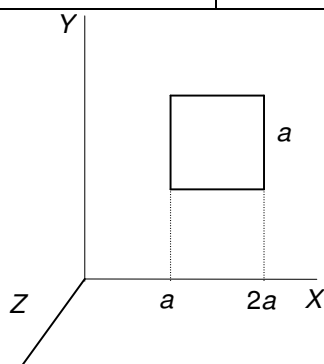


1. (2,5 points) A squared loop with side  $a$  and resistance  $R$  is placed inside a **uniform** and not stationary magnetic field  $\mathbf{B} = (3t + 2)\mathbf{k} \text{ T}$  ( $t$  is the time in seconds). Find:

- Magnetic flux  $\phi$  through the loop, as a function of  $t$ .
- Induced electromotive force  $\epsilon$  on the loop.
- Intensity of current  $i$  flowing along the loop, giving its direction.
- Force  $\mathbf{F}$  (in vector form) acting on the upper side of the loop when  $t=2 \text{ s}$ .
- Explain (calculations are not needed) if the total force acting on the whole loop should be null (or not).

1. (2,5 puntos) Una espira cuadrada de lado  $a$  y resistencia  $R$  se encuentra dentro de un campo magnético uniforme y no estacionario  $\mathbf{B} = (3t + 2)\mathbf{k} \text{ T}$  ( $t$  es el tiempo en segundos). Calcula:

- Flujo magnético  $\phi$  que atraviesa la espira en función de  $t$ .
- Fuerza electromotriz  $\epsilon$  inducida en la espira.
- Intensidad de corriente  $i$  que circula por la espira, indicando su sentido.
- Fuerza  $\mathbf{F}$  (en forma vectorial) que actúa sobre el lado superior de la espira en el instante  $t=2 \text{ s}$ .
- Razona (sin hacer los cálculos) si la fuerza total que actúa sobre la espira completa debe ser nula, o no.



### Solution

a) As magnetic field is uniform and parallel to surface vector:  $\phi = \int_S \mathbf{B} \cdot d\mathbf{S} = \int_S B \cdot d\mathbf{S} = B \int_S dS = BS = (3t + 2)a^2$

b)  $\epsilon = \frac{d\phi}{dt} = 3a^2$

c)  $i = \frac{\epsilon}{R} = \frac{3a^2}{R}$  The magnetic field is exiting from the paper, and then the exiting flux is increasing on time. Therefore, the induced current must create a magnetic field (and a flux) entering on paper. To do it, the induced current must be clockwise.

d)  $B(t=2) = (3 \cdot 2 + 2)\mathbf{k} = 8\mathbf{k} \text{ T}$

As magnetic field is uniform:  $\mathbf{F}(t=2) = i(L \times \mathbf{B}(t=2)) = \frac{3a^2}{R} \begin{vmatrix} i & j & k \\ a & 0 & 0 \\ 0 & 0 & 8 \end{vmatrix} = \frac{24a^3}{R}(-j)$

e) The total force acting on the whole loop will be null because the force acting on a loop inside a magnetic field is zero if the magnetic field is uniform. As  $\mathbf{B}$  is uniform, then the total force acting on the loop will be null.

2. (2,5 points) Let's consider a coil having **50 cm** length, **3000 turns** and radius **20 cm**, flowed by an intensity of current **2 A**. A second coil with the same length, **400 turns**, and radius **5 cm** is coaxially placed inside the first one. Compute:

- The magnetic field produced by the first coil at a point of its axis.
- The flux through the second coil produced by the first one.
- The mutual inductance coefficient between both coils.
- If the current along first coil is varying on time according  $i(t)=2\cos(100t)$ , compute the absolute value of the electromotive force induced on the second coil.

2. (2,5 puntos) Sea un solenoide de **50 cm** de longitud, **3000 espiras**, y **20 cm** de radio, por el que circula una corriente de **2 A**. Un segundo solenoide de la misma longitud, **400 espiras** y **5 cm** de radio está situado coaxialmente dentro del primero. Calcular:

- El campo magnético producido por el primer solenoide en un punto de su eje.
- El flujo que el primer solenoide produce sobre el segundo.
- El coeficiente de inducción mutua entre ambos solenoides.
- Si la corriente en el primer solenoide varía con el tiempo según la expresión  $i(t)=2\cos(100t)$ , calcula el valor absoluto de la f.e.m. inducida en el segundo solenoide.

Solution

a) By considering the magnetic field uniform inside the coil  $B = \mu_0 \frac{3000}{50 \cdot 10^{-2}} 2 = 12\mu_0 10^3 = 48\pi 10^{-4} T$

b)  $\phi = BNS = 12\mu_0 10^3 400\pi (5 \cdot 10^{-2})^2 = 12\mu_0 \pi 10^3 = 48\pi^2 10^{-4} Wb$

c)  $M = \frac{\phi}{I} = \frac{12\mu_0 \pi 10^3}{2} = 6\mu_0 \pi 10^3 = 24\pi^2 10^{-4} H$

d)  $|\varepsilon_2| = \left| \frac{d\phi_2}{dt} \right| = \left| \frac{d(Mi)}{dt} \right| = \left| M \frac{di}{dt} \right| = \left| 24\pi^2 10^{-4} (100 \cdot 2 \cdot \text{sen}(100t)) \right| = \left| 48\pi^2 10^{-2} \text{sen}(100t) \right| V$

3. (2,5 points) A capacitor **2,5 mF** sized and a **3 Ω** resistor are connected in series (dipole RC). Along the RC dipole flows a current  $i(t)=5\cos(100t-10^\circ)$

A. Find:

- The instantaneous voltage on resistor,  $u_R(t)$ .
- The instantaneous voltage on capacitor,  $u_C(t)$ .
- The impedance, Z, and the phase lag of dipole,  $\varphi$ . Draw the impedance triangle.
- The instantaneous voltage on terminals of RC dipole,  $u(t)$ .
- Compute the self-inductance coefficient, L, of the inductor should be added in series to R and C in order the new circuit was in resonance (at the given angular speed).

3. (2,5 puntos) Un condensador de **2,5 mF** y una resistencia de **3 Ω** están conectados en serie (dipolo RC). Por el dipolo RC circula una corriente  $i(t)=5\cos(100t-10^\circ)$

A. Calcula:

- La tensión instantánea en la resistencia,  $u_R(t)$ .
- La tensión instantánea en el condensador,  $u_C(t)$ .
- La impedancia, Z, y el ángulo de desfase del dipolo,  $\varphi$ . Dibuja el triángulo de impedancias.
- La tensión instantánea en los terminales del dipolo RC,  $u(t)$ .
- Calcula el coeficiente de autoinducción, L, que debería tener una autoinducción añadida en serie con R y C, de tal manera que el nuevo circuito esté en resonancia (con la pulsación dada).

Solution:

a)  $u_R(t) = 3 \cdot 5 \cos(100t - 10^\circ) = 15 \cos(100t - 10^\circ) V$

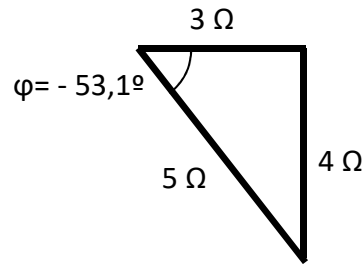
b)  $X_C = \frac{1}{C\omega} = \frac{1}{2,5 \cdot 10^{-3} \cdot 100} = 4 \Omega$

$-90 = \varphi_u - (-10) \Rightarrow \varphi_u = -90 - 10 = -100^\circ$

$u_C(t) = 4 \cdot 5 \cos(100t - 100^\circ) = 20 \cos(100t - 100^\circ) V$

c)  $Z = \sqrt{R^2 + (1/C\omega)^2} = \sqrt{3^2 + 4^2} = 5 \Omega$        $\text{tg}\varphi = \frac{-X_C}{R} = \frac{-4}{3} = -1,33 \Rightarrow \varphi = -53,1^\circ = -0,93 \text{rad}$

The impedance triangle is:



d)  $-53,1 = \varphi_u - (-10) \Rightarrow \varphi_u = -53,1 - 10 = -63,1^\circ$        $U_m = I_m Z = 5 \cdot 5 = 25 \text{ V}$

$u(t) = 25 \cos(100t - 63,1^\circ) \text{ V}$

e) A RLC circuit is in resonance when  $X_L = X_C$ . As  $X_C = 4 \Omega$ , must be verified:

$X_L = L\omega = X_C \Rightarrow L \cdot 100 = 4 \Rightarrow L = \frac{4}{100} = 0,04 \text{ H} = 40 \text{ mH}$

4. (2,5 points) On circuit on picture every diodes have drop forward voltage  $V_u = 0,7 \text{ V}$ , and internal resistance negligible (also the battery).

a) On a table like the next one, mark with a cross the correct bias (forward or reverse) for every diode:

Diode	Forward	Reverse
D <sub>1</sub>		
D <sub>2</sub>		
D <sub>3</sub>		

b) Compute intensities  $I_1$ ,  $I_2$  and  $I_3$  flowing along diodes D<sub>1</sub>, D<sub>2</sub> and D<sub>3</sub>.

c) Compute the differences of potential  $V_A - V_C$  and  $V_C - V_B$ .

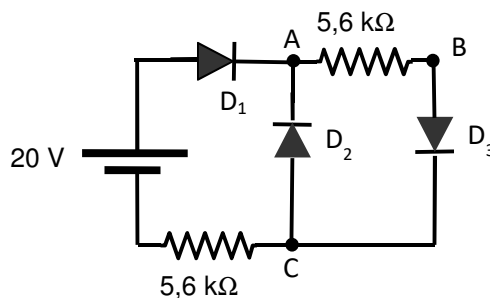
4. (2,5 puntos) En el circuito de la figura, todos los diodos tienen tensión umbral  $V_u = 0,7 \text{ V}$ , y resistencia interna despreciable (también el generador).

a) En una tabla como la siguiente, marca con una cruz la correcta polarización de cada diodo:

Diodo	Directa	Inversa
D <sub>1</sub>		
D <sub>2</sub>		
D <sub>3</sub>		

b) Calcula las intensidades  $I_1$ ,  $I_2$  e  $I_3$  que circulan por los diodos D<sub>1</sub>, D<sub>2</sub> y D<sub>3</sub>.

c) Calcula las diferencias de potencial  $V_A - V_C$  y  $V_C - V_B$ .



Solution:

a)

Diode	Forward	Reverse
D <sub>1</sub>	X	
D <sub>2</sub>		X
D <sub>3</sub>	X	

$$b) I_1 = I_3 = \frac{20 - 0,7 - 0,7}{5,6 + 5,6} = 1,66 \text{ mA} \quad I_2 = 0$$

$$c) \text{ As } D_2 \text{ is reverse biased } V_A - V_C = I_3 \cdot 5,6 + 0,7 = 1,66 \cdot 5,6 + 0,7 = 10 \text{ V}$$

On the other hand, as  $D_3$  is forward biased,  $V_C - V_B = -0,7 \text{ V}$

## FORM

**Magnetic Forces**  $F = q(v \times B)$   $dF = Idl \times B$   $\mu = N \cdot I \cdot S$   $\tau = \mu \times B$   $V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$

**Sources of magnetic field**  $dB = \frac{\mu_0 I d \times r}{4\pi r^3}$   $\mu_0 = 4\pi 10^{-7} \text{ (I.S.units)}$   $B = \frac{\mu_0 I}{2\pi x}$

$$B = \frac{\mu_0 I}{2R} \quad \int_L B \cdot dl = \mu_0 \sum I \quad B = \frac{\mu_0 NI}{l}$$

**Electromagnetic induction**  $|\varepsilon| = \frac{d\phi}{dt}$   $\phi = L \cdot I$   $\phi_{21} = M \cdot I_1$   $W_L = \frac{1}{2} L \cdot I^2$

**Alternating current**  $\varphi = \varphi_u - \varphi_i$   $X_L = L\omega$   $X_C = \frac{1}{C\omega}$   $U_{rms} = \frac{U_m}{\sqrt{2}}$   $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$tg\varphi = \frac{L\omega - 1/C\omega}{R} \quad Z = \frac{U_m}{I_m} = \sqrt{R^2 + (L\omega - 1/C\omega)^2}$$

$$P(t) = u(t) \cdot i(t) = U_m I_m \cos\varphi \sin^2 \omega t + \frac{U_m I_m}{2} \sin\varphi \sin 2\omega t \quad P = U_{rms} I_{rms} \cos\varphi \quad f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

**Semiconductors**  $n \cdot p = n_i^2$   $N_A + n = N_D + p$