

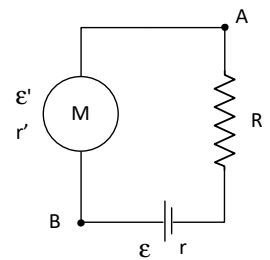


1. (2,5 points) On circuit on picture, it's known that the powers consumed by **Joule heating** on internal resistors of **motor** and **generator** and on **resistor R** is **10 W**:  $P_R=P_r=P_{r'}=10\text{ W}$ . Moreover, the difference of potential between terminals of resistor R is 5 V:  $V_R=5\text{ V}$ , and the **efficiency of motor is 80%**. Find (0,5 points each):

- The **turned power** on mechanical energy by the motor and the **generated power** by the generator. State the **polarity** of motor.
- Compute the **intensity** flowing along the circuit.
- Compute the features of motor, generator and resistor:  $\epsilon$ ,  $r$ ,  $\epsilon'$ ,  $r'$  and  $R$ .
- Compute the **difference of potential** between points **A** and **B**.
- Compute the **efficiency of generator**. **Explain** if this **efficiency** could be or not, equal or less than that of motor.

1. (2,5 puntos) En el circuito de la figura, se sabe que la potencias consumidas por **efecto Joule** en las resistencias internas de **motor** y **generador** y en la **resistencia R** son **10 W**:  $P_R=P_r=P_{r'}=10\text{ W}$ . Además, la diferencia de potencial entre los terminales de la resistencia R es 5 V:  $V_R=5\text{ V}$ , y el **rendimiento del motor es del 80%**. Calcula (0,5 puntos cada apartado):

- La **potencia transformada** por el motor en energía mecánica y la **generada** por el generador. Indica la **polaridad** del motor.
- Calcula la **intensidad** que recorre el circuito.
- Calcula las características de motor, generador y resistencia:  $\epsilon$ ,  $r$ ,  $\epsilon'$ ,  $r'$  y  $R$ .
- Calcula la **diferencia de potencial** entre los puntos **A** y **B** del circuito.
- Calcula el **rendimiento del generador**. **Razona** si este **rendimiento** podría, o no, ser menor o igual que el del motor.



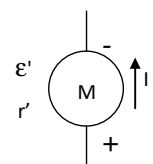
**Solution:**

$$a) \text{ From the efficiency of motor } \eta' = \frac{P_t}{P_c} = \frac{P_c - P_{r'}}{P_c} = \frac{P_c - 10}{P_c} = 0,8 \Rightarrow P_c = 50\text{ w}$$

$$\text{Therefore } P_t = P_c - P_{r'} = 50 - 10 = 40\text{ w}$$

Related to the generator, the power supplied to the circuit is the sum of that consumed by the motor and that consumed by the resistor R:  $P_s = P_c + P_R = 50 + 10 = 60\text{ w}$  And therefore  $P_g = P_s + P_r = 60 + 10 = 70\text{ w}$

According the polarity of generator, the intensity will flow in clockwise direction, being the polarity of motor that shown on picture:



- If it's known the consumed power on R and the difference of potential between its terminals, then R can be calculated  $P_R = 10 = \frac{V_R^2}{R} = \frac{5^2}{R} \Rightarrow R = \frac{25}{10} = 2,5\ \Omega$

$$\text{And then } V_R = IR \Rightarrow I = \frac{V_R}{R} = \frac{5}{2,5} = 2\text{ A}$$

- As the consumed powers on  $r$ ,  $r'$  and R are equal and the intensity is the same for every resistor, then these resistors are equal. Therefore  $r = r' = R = 2,5\ \Omega$

$$\text{About } \epsilon \text{ and } \epsilon': P_g = 70 = \epsilon I \Rightarrow \epsilon = \frac{P_g}{I} = \frac{70}{2} = 35\text{ V} \quad P_t = 40 = \epsilon' I \Rightarrow \epsilon' = \frac{P_t}{I} = \frac{40}{2} = 20\text{ V}$$

- If we travel from A to B along the motor  $V_{AB} = -r'I - \epsilon' = -2 \cdot 2,5 - 20 = -25\text{ V}$

Obviously, along the generator, the result is the same  $V_{AB} = I(R+r) - \epsilon = 2(2,5+2,5) - 35 = -25\text{ V}$

e)  $\eta = \frac{P_s}{P_g} = \frac{60}{70} \approx 86\%$

This efficiency of generator can never be equal or less than that of motor, because it would mean that the power generated by the generator would be equal or less than that consumed by the motor (remind that the power consumed on both internal resistors is the same). That's impossible because the resistor R must consume some power, and then we can conclude that it is impossible that the efficiency of generator can be equal or less than that of motor:

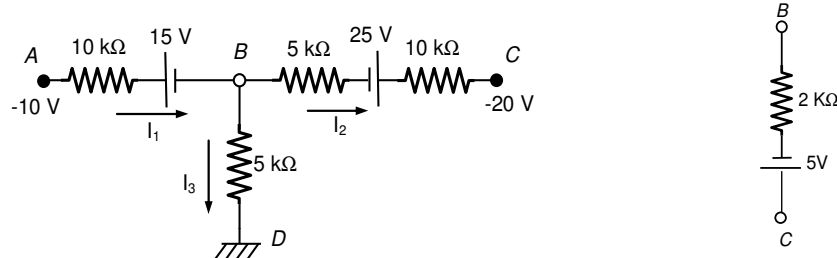
$$\left. \begin{aligned} \eta_g &= \frac{P_s}{P_g} = \frac{P_g - P_r}{P_g} = 1 - \frac{P_r}{P_g} \\ \eta_m &= \frac{P_t}{P_c} = \frac{P_c - P_{r'}}{P_c} = 1 - \frac{P_{r'}}{P_c} \end{aligned} \right\} \text{As } P_r = P_{r'}, \text{ if } \eta_g \leq \eta_m \Rightarrow -\frac{P_r}{P_g} \leq -\frac{P_{r'}}{P_c} \Rightarrow P_c \geq P_g \text{ But that's impossible, as we said above.}$$

2. (3 points) Given the circuit on picture, compute:

- (1) Intensity of current flowing along each branch with the shown directions,  $I_1$ ,  $I_2$  and  $I_3$ . Say if batteries of 15 and 25 V are acting as generators or receptors.
- (1) Thevenin's equivalent generator between points B and C, clearly showing its polarity.
- (0,5) A new branch (that shown on right) is connected between points B and C. Is the 5 V generator of the new branch, generating or consuming energy? Compute its generated or consumed power.
- (0,5) Thevenin's equivalent generator between points A and D, clearly showing its polarity.

2. (3 puntos) Dado el circuito de la figura, calcula:

- (1) La intensidad de corriente en cada rama con los sentidos mostrados,  $I_1$ ,  $I_2$  y  $I_3$ . Indica si los elementos de 15 y 25 V actúan como generadores o receptores.
- (1) El generador equivalente de Thevenin entre los puntos B y C, indicando claramente su polaridad.
- (0,5) Al circuito se le añade una nueva rama (la mostrada a la derecha) entre los puntos B y C. Indica si el elemento de 5 V de la nueva rama, genera o consume energía y calcula su valor.
- (0,5) El generador equivalente de Thevenin entre los puntos A y D, indicando claramente su polaridad.



Solution:

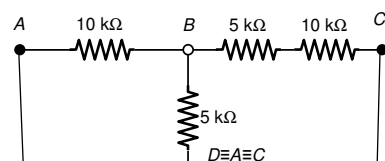
This is a network with 2 junctions and two loops, and so we'll need one equation for junctions and two equations for loops:

$$\left. \begin{aligned} I_1 &= I_2 + I_3 \\ a) \quad V_{AD} &= -10 = 10I_1 + 15 + 5I_3 \\ V_{CD} &= -20 = -10I_2 + 25 - 5I_2 + 5I_3 \end{aligned} \right\} \Rightarrow I_1 = -1 \text{ mA} \quad I_2 = 2 \text{ mA} \quad I_3 = -3 \text{ mA}$$

According the computed intensities, 15 V and 25 V batteries are acting as generators.

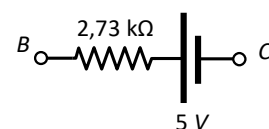
b)  $\mathcal{E}_T = V_{BC} = V_B - V_C = (5 + 10)I_2 - 25 = 15 \cdot 2 - 25 = 5 \text{ V}$

Passive circuit after removing all the generators is



and its equivalent resistance between B and C:

$$\frac{1}{R_{eqBC}} = \frac{1}{10} + \frac{1}{5} + \frac{1}{5+10} \Rightarrow R_{eqBC} = \frac{30}{11} \approx 2,73 \text{ k}\Omega$$

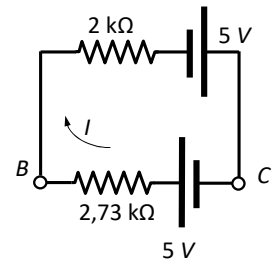


So, Thevenin's equivalent generator between B and C is:

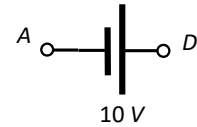
- c) If we connect the new branch between points B and C, the resulting circuit is that on picture. The intensity flows in clockwise direction, with a magnitude:

$$I = \frac{5+5}{2,73+2} = 2,11 \text{ mA}$$

The 5 V battery of the new branch is generating energy. The generated power is  $P_g = \varepsilon I = 5 \cdot 2,11 = 10,55 \text{ mW}$



- d)  $\varepsilon_T = V_{AD} = -10 \text{ V}$  And  $R_{eqAD} = 0$  Therefore, the Thevenin's equivalent generator between A and D is

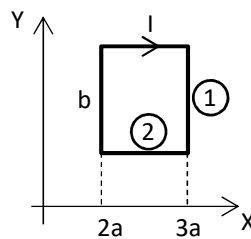


**3. (2 points)** Let's consider a rectangular loop with sides  $a$  and  $b$ , flowed by an intensity of current  $I$  in the given direction. The loop is placed inside a **no uniform** magnetic field  $\vec{B} = B_0 \frac{a}{x} \vec{k}$ . Compute:

- The **magnetic force** acting on sides **1** and **2**.
- (0,5) The **magnetic moment**  $\vec{\mu}$  of the loop.
- (0,5) If the magnetic field was  $\vec{B} = B_0 \vec{k}$  ( $B_0$  a positive constant), compute the **torque**  $\vec{\tau}$  of the magnetic forces acting on the loop.

**3. (2 puntos)** Sea la **espira** rectangular de la figura de lados  $a$  y  $b$ , recorrida por una corriente de intensidad  $I$  en el sentido indicado, situada en el interior de un campo magnético **no uniforme** de valor  $\vec{B} = B_0 \frac{a}{x} \vec{k}$ . Calcula:

- La **fuerza magnética** que aparece sobre los lados **1** y **2**.
- (0,5) El **momento magnético**  $\vec{m}$  de la espira.
- (0,5) Si el campo magnético fuera  $\vec{B} = B_0 \vec{k}$  ( $B_0$  una constante positiva), calcula el **momento**  $\vec{M}$  de las fuerzas magnéticas que actúan sobre la espira.



**Solution:**

$$a) \vec{F}_1 = I(-b)\vec{j} \times B_0 \frac{a}{3a} \vec{k} = -\frac{IbB_0}{3} \vec{i} \quad \vec{F}_2 = \int_{2a}^{3a} I(-dx\vec{i}) \times B_0 \frac{a}{x} \vec{k} = IB_0 a \int_{2a}^{3a} \frac{dx}{x} \vec{j} = IB_0 a \ln \frac{3}{2} \vec{j}$$

$$b) \text{ As we have only one loop: } \vec{\mu} = I\vec{S} = -Iab\vec{k}$$

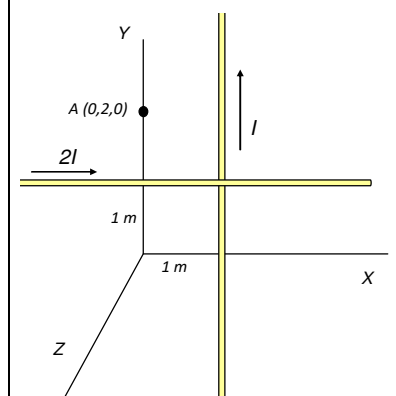
$$\vec{\tau} = \vec{\mu} \times \vec{B} = -Iab\vec{k} \times B_0\vec{k} = 0$$

**4. (2,5 points)** Two **infinite** straight carrying current **conductors** are placed on **plane XY**, parallel to axes X and Y respectively, at a distance **1 m** each, as can be seen on picture. Their intensities are **2I** and **I**. Find:

- The **magnetic field vector** created by both currents at point **A (0,2,0)** m.
- A point **P** lying over the **X axis** where the **magnetic field was null**. Give its **coordinates**.
- A **circular loop** (radius **0,5 m**) flowed by an intensity  $I'$  is placed on

**4. (2,5 puntos)** Dos **conductores** rectilíneos e **indefinidos** están colocados en el **plano XY**, paralelos a los ejes X e Y respectivamente, a una distancia de **1 m** cada uno, como se puede ver en la figura. Sus intensidades son **2I** e **I**. Halla:

- El **vector campo magnético** producido por ambas corrientes en el punto **A (0,2,0)** m.
- Un punto **P** del **eje X** donde se **anule el campo magnético total**. Da sus **coordenadas**.
- Una **espira circular** (radio **0,5 m**)



<p>the <b>plane XY</b>, being its center the point <b>A</b>. Which should be the <b>direction</b> of <b>I'</b> (clockwise or counterclockwise) in order the <b>magnetic field</b> at point <b>A</b> was <b>null</b>? Find the <b>magnitude</b> of <b>I'</b> in such case.</p>	<p>recorrida por una corriente <b>I'</b> es colocada en el <b>plano XY</b> con su centro en el punto <b>A</b>. ¿Cuál debe ser el <b>sentido</b> de <b>I'</b> (horario o antihorario) para que el <b>campo magnético</b> en <b>A</b> se <b>anule</b>? Calcula el <b>valor</b> que debe tener <b>I'</b> en este caso.</p>	
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**Solution:**

On every case, the magnetic field is the summatory of magnetic field produced by each conductor:

a) 
$$\vec{B}_A = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 2I}{2\pi \cdot 1} \vec{k} + \frac{\mu_0 I}{2\pi \cdot 1} \vec{k} = \frac{3\mu_0 I}{2\pi} \vec{k} = 6I \cdot 10^{-7} \vec{k}$$

b) In order the magnetic fields created by both conductors can be cancelled, such point P must be placed on left of vertical conductor. If the coordinates of such point P are (x,0,0) (x<1), then must be verified that:

$$\frac{\mu_0 2I}{2\pi \cdot 1} = \frac{\mu_0 I}{2\pi \cdot (1-x)} \Rightarrow 2(1-x) = 1 \Rightarrow x = \frac{1}{2} \text{ m} \quad P(0,5, 0, 0)$$

c) The magnetic field calculated at point A goes out of the page, and then the magnetic field created by the circular loop must go into the page. To do it, I' must be clockwise. And its magnitude must verify:

$$\frac{\mu_0 I'}{2 \cdot 0,5} = 6I \cdot 10^{-7} \Rightarrow I' = \frac{6I \cdot 10^{-7}}{\mu_0} = \frac{6I}{4\pi} = \frac{3I}{2\pi}$$

**FORM**

**Direct current**

$$V_A - V_B = I \sum R - \sum \mathcal{E} \quad I = \frac{\sum \mathcal{E}}{\sum R} \quad P = V \cdot I \quad \mathcal{E} = \frac{dW}{dq} \quad P_R = I^2 \cdot R$$

$$P_g = \mathcal{E} \cdot I \quad P_t = \mathcal{E}' \cdot I \quad P_g - P_r = P_s \quad P_t + P_{r'} = P_c \quad \eta_g = \frac{P_s}{P_g} \quad \eta_r = \frac{P_t}{P_c}$$

**Magnetic Forces**

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad d\vec{F} = Id\vec{l} \times \vec{B} \quad \vec{\mu} = N \cdot I \cdot \vec{S} \quad \vec{\tau} = \vec{\mu} \times \vec{B} \quad V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$$

**Sources of magnetic field**

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3} \quad \mu_0 = 4\pi 10^{-7} \text{ (I.S.units)} \quad B = \frac{\mu_0 I}{2\pi x}$$

$$B = \frac{\mu_0 I}{2R} \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I \quad B = \frac{\mu_0 NI}{l}$$