Retaking First exam FFI
Recuperación 1er parcial FFI
January 24th 2020
Year-Curso 2019/20

Applied Physics Dept.
Dep. de Física Aplicada

1. (2,5 points) Two point charges with **20 nC** and **10 nC** are placed on vacuum at points **(3,-3) m** and **(3,3) m** respectively. Find:

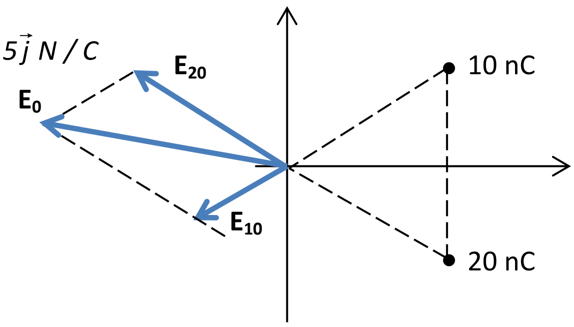
a) The total electric field at point **O (0,0) m**. **Draw** the field created by each charge and the resulting one.
b) The coordinates of a **poin**t where the **electric field** was **null**.
c) Electric **potential** at points **O (0,0) m** and **A (0,3) m**.
d) **The work** done by the electric field to carry a **-2 nC** charge from point **A** to **O.** Say if the charge will spontaneously move towards point O.
e) What is the **electrostatic potential energy** of this system of **charges**?

1. (2,5 puntos) Dos cargas puntuales de **20 nC** y **10 nC** se encuentran en el vacío en los puntos **(3,-3) m** y **(3,3) m** respectivamente. Calcula:

a) El campo eléctrico resultante en el punto **O (0,0) m**. **Dibuja** el campo que crea cada carga y el resultante.
b) Las coordenadas de un **punto** donde el **campo eléctrico** se **anule.**
c) **Potencial** eléctrico en los puntos **O (0,0) m** y **A (0,3) m**.
d) **El trabajo** para llevar una carga de **-2 nC** desde el punto **A** al **O**. Indica si la carga se desplazará espontáneamente hacia el punto O.
e) Cuál es la **energía potencial electrostática** de este sistema de cargas?

Solution:

a) $\vec{E}_O = \frac{9\cdot10^9\cdot10\cdot10^{-9}}{(3\sqrt{2})^2}\left(\frac{-\vec{i}-\vec{j}}{\sqrt{2}}+2\frac{-\vec{i}+\vec{j}}{\sqrt{2}}\right)=\frac{5}{\sqrt{2}}(-3\vec{i}+\vec{j})=-10,6\vec{i}+3,5\vec{j}\ N/C$



b) The electric field can only be null at points along the line joining q_1 and q_2. If we name P this point and its coordinates are P(3,x), then must be verified:

$\frac{kq_2}{(3-x)^2}=\frac{kq_1}{(3+x)^2}\Rightarrow x=9-6\sqrt{2}=0,51\,m$ P(3,0,51) m

c) $V_O=9\cdot10^9\frac{10\cdot10^{-9}}{3\sqrt{2}}(1+2)=\frac{90}{\sqrt{2}}=63,64\,V$ $V_A=9\cdot10^9\cdot10\cdot10^{-9}\left(\frac{1}{3}+\frac{2}{\sqrt{45}}\right)=90\left(\frac{1}{3}+\frac{2}{3\sqrt{5}}\right)=30\left(1+\frac{2}{\sqrt{5}}\right)=56,83\,V$

d) $W_{AO}=q(V_A-V_O)=-2\cdot10^{-9}(56,83-63,64)=13,62\cdot10^{-9}\ J$

As this work is positive, it's done by the forces of the electric field.

e) $W=q_1V_1+q_2V2=q_1k\frac{q_2}{6}+q_2k\frac{q_1}{6}=\frac{2kq_1q_2}{6}=\frac{9\cdot10^9\cdot20\cdot10^{-9}\cdot10\cdot10^{-9}}{3}=6\cdot10^{-7}\ J$

2. (2,5 points) A **sphere** with radius **R** is uniformly charged with a surface density of charge **σ**.

a) Find the **modulus of the electric field** at a distance ***r*** from the centre, **inside** (*r*<R) and **outside** (*r*>R).
b) **Draw,** approximately, a graph where appears the electric field as a function of the distance to the centre. What is the value of the **electric field** at points where **r=R**?
c) Compute the **electric potential** at the **centre** of the sphere.

2. (2,5 puntos) Una **esfera** de radio **R**, está uniformemente cargada con una densidad superficial de carga **σ**.

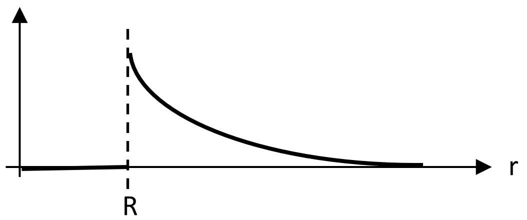
a) Calcular el **módulo del campo eléctrico** a una distancia ***r*** del centro, en el **interior** (*r*<R) y en el **exterior** (*r*>R).
b) **Dibuja,** de forma aproximada, una gráfica donde se represente el campo eléctrico en función de la distancia al centro. ¿Cuánto vale el **campo** en **r=R**?
c) Calcula el **potencial eléctrico** en el **centro** de la esfera.

Solution:

a) Let's consider a spherical surface with radius r<R. As there isn't any charge inside this sphere, then applying gauss's law E=0.

For r>R, if we consider a spherical surface with radius r: $\phi=\int_{Sphere}\vec{E}d\vec{S}=E4\pi r^2=\frac{\sigma4\pi R^2}{\varepsilon_0}\Rightarrow E=\frac{\sigma R^2}{\varepsilon_0 r^2}$

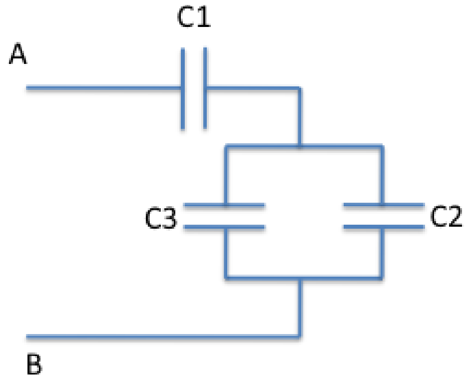
b) The electric field at points where r=R is undefined. The electric field is a discontinuous function, showing a discontinuity at points where some charge exists. Therefore, the electric when r=R is not defined.



c) As the electric field is null inside the sphere, the electric potential at its centre will be equal to that on the surface of the sphere:

$$V_0 = V_{surface} = \int_R^\infty \frac{\sigma R^2}{\varepsilon_0 r^2} dr = \frac{\sigma R}{\varepsilon_0}$$

3. (2,5 points) Three capacitors are associated as can be seen on picture, being $\mathbf{C_2=C_3=2\ \mu F}$. a) ¿Which should be the magnitude of $\mathbf{C_1}$ in order the **equivalent capacitance** of the set was **2 μF**? b) If a difference of potential of **300 V** is applied between points **A and B**, find the **charge** and the **difference of potential** on each **capacitor**. c) The **plates** of capacitor $\mathbf{C_1}$ are moved away at a distance **twice** of the initial. Compute the **total energy** stored in the set of capacitors.	**3. (2,5 puntos) Tres condensadores** se asocian como se indica en la figura, siendo $\mathbf{C_2=C_3=2\ \mu F}$. a) ¿Cuánto debe valer $\mathbf{C_1}$ para que la **capacidad equivalente** del conjunto sea **2 μF**? b) Si se aplica entre los puntos **A** y **B** una diferencia de potencial de **300 V**, encuentra la **carga** y la **diferencia de potencial** de cada **condensador**. c) Las **placas** del condensador $\mathbf{C_1}$ se separan a una distancia **doble** de la inicial. Calcula la **energía total** almacenada en el sistema de condensadores.



Solution:

a) According the rules for the association of capacitors, $\frac{1}{2} = \frac{1}{2+2} + \frac{1}{C_1} \Rightarrow C_1 = 4\ \mu F$

b) $V_{AB} = 300 = \frac{Q_1}{C_{eq}} = \frac{Q_1}{2} \Rightarrow Q_1 = 600\ \mu C$ As C_2 and C_3 are equal $Q_2 = Q_3 = \frac{600}{2} = 300\ \mu C$

$V_1 = \frac{Q_1}{C_1} = \frac{600}{4} = 150\,V \qquad V_2 = V_3 = \frac{300}{2} = 150\,V$

c) If the plates of C_1 are moved away, then the capacitance is halved: $C'_1 = \frac{C_1}{2} = 2\ \mu F$ and the new equivalent capacitance is:

$$\frac{1}{C'_{eq}} = \frac{1}{2+2} + \frac{1}{2} = \frac{3}{4} \Rightarrow C'_{eq} = \frac{4}{3}\ \mu F$$

The charge of the equivalent capacitor when 300 V are applied is: $Q' = C'_{eq}\,300 = \frac{4}{3}300 = 400\ \mu C$

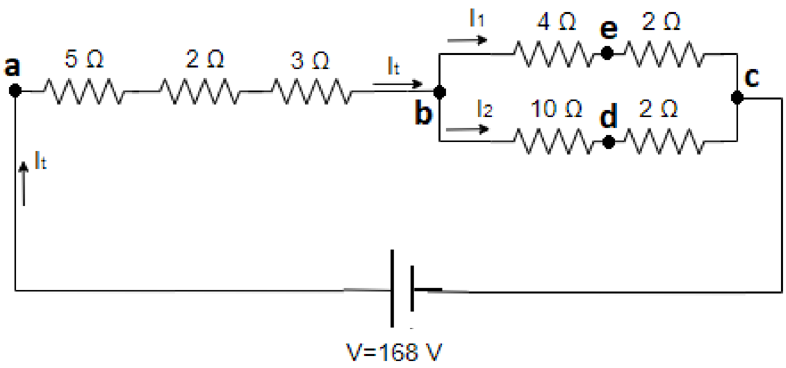
And the total energy stored on the set of capacitors: $W = \frac{Q'^2}{2C'_{eq}} = \frac{400^2 \cdot 10^{-12}}{2\frac{4}{3} \cdot 10^{-6}} = 6 \cdot 10^{-2}\ J$

4. (2,5 points) The set of resistors on picture is connected to an ideal power supply of **168 V**. Compute:

a) The **equivalent resistance** of set between **a** and **c**.
b) **Intensities** of current I_t, I_1, I_2.
c) V_{ab}, V_{bc}, and V_{ed}.

4. (2,5 puntos) El conjunto de resistencias de la figura se conecta a un generador ideal de **168 V**. Calcula:

a) Resistencia equivalente entre **a** y **c.**
b) **Intensidades** de corriente I_t, I_1, I_2.
c) V_{ab}, V_{bc}, y V_{ed}.



Solution:

a) $\frac{1}{R_{bc}} = \frac{1}{4+2} + \frac{1}{10+2} = \frac{3}{12} \Rightarrow R_{bc} = \frac{12}{3} = 4\,\Omega$ $\qquad R_{ac} = 5+2+3+4 = 14\,\Omega$

b) $I_t = \frac{168}{14} = 12\,A$ $\qquad V_{ab} = I_t 10 = 12\cdot 10 = 120\,V$ $\qquad V_{bc} = 168 - 120 = 48\,V$

$I_1 = \frac{48}{6} = 8\,A$ $\qquad I_2 = \frac{48}{12} = 4\,A$

c) V_{ab} and V_{bc} have already been computed. Related to V_{ed}:

$V_{ed} = -I_1 4 + I_2 10 = -8\cdot 4 + 4\cdot 10 = 8\,V$

Si te examinas sólo de una parte, debes resolver los cuatro problemas de esa parte.
If you are sitting only one part of the exam, then you have to solve the four problems of that part.

Si te examinas de dos partes: *1 and 2: 1,2,3,5,6,7* *1 and 3: 1,2,3,9,11,12* *2 and 3: 5,6,7,9,11,12*
If you are sitting two parts of the exam: 1 and 2: 1,2,3,5,6,7 *1 and 3: 1,2,3,9,11,12* *2 and 3: 5,6,7,9,11,12*

Si te examinas de las tres partes, debes resolver: *1,3,6,7,9,11*
If you are sitting the three parts of the exam, then you have to solve: 1,3,6,7,9,11

Form – Fórmulas

Electrostatics

$\vec{F} = k\frac{q_1 q_2}{r^2}\vec{u}_r$ $\qquad \vec{E} = \frac{\vec{F}}{q}$ $\qquad K = \frac{1}{4\pi\varepsilon_0} = 9\cdot 10^9 (S.I.)$ $\qquad V_A - V_B = \int_A^B \vec{E}\cdot \vec{dr}$

$\vec{E} = K\frac{q}{r^2}\vec{u}_r$ $\qquad V = K\frac{q}{r}$ $\qquad \int_S \vec{E}\cdot \vec{dS} = \frac{\sum Q}{\varepsilon_0}$ $\qquad W_{AB} = q(V_A - V_B)$

Conductors and capacitors $\qquad E = \frac{\sigma}{\varepsilon_0}$ $\qquad C = \frac{Q}{V}$ $\qquad C = \frac{\varepsilon_0 S}{d}$

$C_{eq} = \sum C_i$ $\qquad \frac{1}{C_{eq}} = \sum \frac{1}{C_i}$ $\qquad E_d = \frac{E}{\varepsilon_r}$ $\qquad C_d = \varepsilon_r C$ $\qquad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$

Direct Current $\qquad \vec{J} = n\cdot e\cdot \vec{v}_d$ $\qquad \vec{J} = \sigma\vec{E}$ $\qquad R = \frac{V_1 - V_2}{I}$ $\qquad R = \rho\frac{L}{S}$

$\rho = \rho_0(1+\alpha(T-T_0))$ $\qquad P_R = IV = I^2 R = \frac{V^2}{R}$