9. (2,5 points) The loop on picture is placed on plane YZ, as can be seen on picture. The side ***AA'***, with length ***h=2 m*** and resistance ***R=1 Ω*** is moving with constant speed ***v=1m/s*** in the positive direction of OY axis, being $\ell = vt$. Over the loop acts a magnetic field $\vec{B} = 2y\vec{i} + 3\vec{j} + 4\vec{k}\ T$ (y in m).

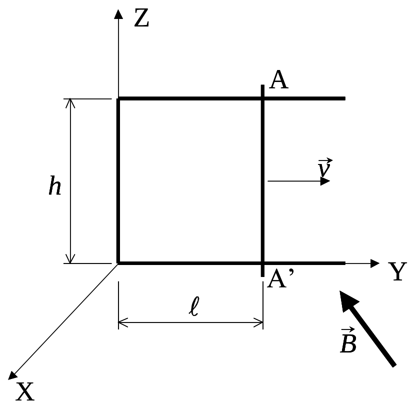
Determine, for the time ***t=1s***:

a) **Magnetic flux** ϕ through the loop.
b) **Induced electromotive force** ε on the loop.
c) **Induced intensity of current i** flowing along the loop, giving its **direction**.
d) Force $\vec{F}$ acting on the moving side.

9. (2,5 puntos) La espira de la figura está situada en el plano YZ, en la posición indicada. El lado ***AA'***, de longitud ***h=2 m*** y resistencia ***R=1 Ω***, se mueve con velocidad constante ***v=1m/s*** en la dirección positiva del eje OY, siendo $\ell = vt$. Sobre la espira actúa un campo magnético de expresión $\vec{B} = 2y\vec{i} + 3\vec{j} + 4\vec{k}\ T$ (y en m).

Determina, para el instante de tiempo ***t=1s***:

a) El **flujo magnético** a través de la espira.
b) La **fuerza electromotriz** inducida.
c) La **intensidad de corriente inducida** y su **sentido**.
d) La **fuerza magnética** sobre el lado móvil.



Solution:

a) On t=1 s $\ell(t=1)=1\,m$ and the flux across the loop (in the direction of positive X axis)

$$\phi(t=1)=\int_S \vec{B}\cdot d\vec{S}=\int_S (2y\vec{i}+3\vec{j}+4\vec{k})hdy\vec{i}=\int_0^{\ell} 2yhdy=2h\left.\frac{y^2}{2}\right|_0^1=2\,Wb \qquad \text{Out of page}$$

b) For a time t $\phi(t)=\int_S \vec{B}\cdot d\vec{S}=\int_S (2y\vec{i}+3\vec{j}+4\vec{k})hdy\vec{i}=\int_0^{\ell} 2yhdy=2h\left.\frac{y^2}{2}\right|_0^{vt}=hv^2t^2$

$$|\varepsilon|=\left|\frac{d\phi}{dt}\right|=\frac{d\phi}{dy}\frac{dy}{dt}=2hv^2t \qquad \text{On time t=1 s} \quad |\varepsilon(t=1)|=2hv^2\cdot 1=4\,V$$

c) $i=\frac{\varepsilon}{R}=\frac{4}{1}=4\,A$ The magnetic flux is exiting from the paper. As the surface of the loop is incrasing, this exiting magnetic flux is also increasing. Therefore, in order to avoid this increasing of magnetic flux, the induced current must be clockwise.

d) The magnetic field at points of moving side on time t=1 is $\vec{B}(y=vt,t=1)=2y\vec{i}+3\vec{j}+4\vec{k}=2\vec{i}+3\vec{j}+4\vec{k}$

The length vector of moving side, according the direction of induced current is: $\vec{L}=-h\vec{k}=-2\vec{k}\ m$

And the force: $\vec{F}=i\vec{L}\times\vec{B}=4\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & -2\\ 2 & 3 & 4\end{vmatrix}=4(6\vec{i}-4\vec{j})=8(3\vec{i}-2\vec{j})\,N$

10. (2,5 points) Along a solenoid with ***80 windings***, ***5 cm*** of diameter, resistance **10 Ω** and length ***50 cm*** flows an intensity of current **3A**. A coaxial coil made up by **10 windings** with the same diameter than the

10. (2,5 puntos) Por un solenoide de ***80 espiras***, ***5 cm*** de diámetro, resistencia de **10 Ω** y ***50 cm*** de longitud circula una intensidad de corriente de **3A**. Una bobina coaxial formada por **10 espiras** del mismo diámetro que el solenoide, rodea la

solenoid, surrounds the cross section of solenoid. The terminals of such coil are connected to an ammeter, being the total resistance of coil, ammeter and wires, **25 Ω**. By assuming that **magnetic field is uniform** inside both solenoids, compute: a) **Magnetic field** created by the first solenoid at a point of its axis. b) F**lux of magnetic field** across the 10 windings coil. c) **Mutual inductance coefficient** between solenoid and coil. d) **Voltage** on terminals of both devices, solenoid and coil. e) **Intensity** flowing along the ammeter if the intensity of current along the solenoid linearly decreases from **3 A to 1 A in 1 s**.	sección recta central del solenoide. Los terminales de esta bobina se conectan a un amperímetro, siendo la resistencia total de bobina, amperímetro y conductores, de **25 Ω**. Admitiendo que el **campo magnético es uniforme** en el interior de los solenoides, calcula: a) El **campo magnético** producido por el primer solenoide en un punto de su eje. b) El **flujo del campo magnético** a través de la bobina de 10 espiras. c) El **coeficiente de inducción mutua** entre solenoide y bobina. d) La **tensión en los terminales** de ambos, solenoide y bobina. e) La **intensidad** que pasa por el amperímetro cuando la intensidad en el solenoide **disminuye linealmente de 3 A a 1 A en 1 s**.

Solution:

a) By considering the magnetic field uniform inside the coil

$$B=\mu_0\frac{80}{50\cdot 10^{-2}}3=\frac{24}{5}\mu_0 10^2=60,3\cdot 10^{-5}\ T$$

b) $\phi=BNS=\frac{24}{5}\mu_0 10^2\cdot 10\cdot\pi(\frac{5}{2}\cdot 10^{-2})^2=3\mu_0\cdot\pi=11,8\cdot 10^{-6}\ Wb$

c) $M=\frac{\phi}{I}=\frac{94,74\cdot 10^{-6}}{3}=31,58\ \mu H$

d) $\varepsilon_s=iR=3\cdot 10=30\,V \qquad \varepsilon_b=0\,V$

e) $\frac{di}{dt}=\frac{3-1}{1}=2\ A/s \qquad \varepsilon_b=\frac{d\phi_b}{dt}=M\frac{di}{dt}=31,58\cdot 10^{-6}\cdot 2=63,16\cdot 10^{-6}\ V$

$$i_b=\frac{\varepsilon_b}{R}=\frac{63,16\cdot 10^{-6}}{25}=2,52\cdot 10^{-6}\ A$$

11. **(2,5 points)** On a RCL series circuit, the self-inductance coefficient is **0´5 H**, the total applied voltage ***u(t) = 400cos(200t) V*** and the intensity of current ***i(t)=4cos(200t-30º)*** A. Compute: a) Magnitude of ***R* and *C*.** b) Instantaneous **voltage** on every device, $u_R(t)$, $u_L(t)$ *and* $u_C(t)$ c) The **frequency** for which the **impedance** of the circuit is the **minimum**. d) Draw the **impedance triangle** for the initial frequency and for that calculated on point c).	**11**. **(2,5 puntos)** En un circuito RCL en serie, el coeficiente de autoinducción es **0´5 H**, la tensión total aplicada ***u(t) = 400cos(200t) V*** y la intensidad ***i(t)=4cos(200t-30º)*** A. Calcular: a) Los valores de ***R* y *C*.** b) **Tensión** instantánea en cada uno de los elementos, $u_R(t)$, $u_L(t)$ *y* $u_C(t)$ c) La **frecuencia** para la cual la **impedancia** del circuito es **mínima**. d) Dibuja el **triángulo de impedancias** para la frecuencia inicial y para la obtenida en el apartado c).

Solution:

a) $X_L=0,5\cdot 200=100\,\Omega$

From phase lag of dipole $\varphi=0-(-30º)=30º\Rightarrow tg(30º)=\frac{1}{\sqrt{3}}=\frac{X_L-X_C}{R}=\frac{100-X_C}{R}\Rightarrow 100-X_C=\frac{R}{\sqrt{3}}$

Moreover $Z=\frac{U_{\max}}{I_{\max}}=\frac{400}{4}=100=\sqrt{R^2+\left(100-X_C\right)^2}=\sqrt{R^2+\frac{R^2}{3}}=\sqrt{\frac{4R^2}{3}}=\frac{2R}{\sqrt{3}}\Rightarrow R=50\sqrt{3}=86,6\,\Omega$

And $X_C=100-\frac{R}{\sqrt{3}}=100-\frac{50\sqrt{3}}{\sqrt{3}}=50\,\Omega=\frac{1}{C\omega}\Rightarrow C=\frac{1}{50\cdot 200}=10^{-4}\ F=100\ \mu F$

b) $u_R(t)=200\sqrt{3}\cos(200t-30º)=346,4\cos(200t-30º)\,V$

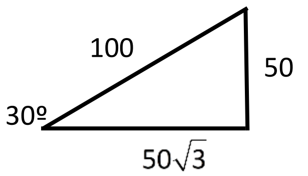
$u_L(t)=400\cos(200t+60º)\,V$

$u_C(t)=200\cos(200t-120º)\,V$

c) The impedance is minimum at the resonant frequency, f_0:

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}=\frac{1}{2\pi}\sqrt{\frac{1}{0,5\cdot 10^{-4}}}=22,5\,Hz$$

d) Initial frequency (31,8 Hz) Resonant frequency (22,5 Hz)



$50\sqrt{3}$

12. **(2,5 points)** A semiconductor built with Germanium, at 300 K, is **dopped** with **2·10^22 átoms of Arsenic/m^3 (donor** atom) (n_i=2,36·10^19 m^-3 for Germanio at 300K)

a) Compute the **density** of **electrons and holes** on such semiconductor.

If the **doping** is **2·10^20 atoms** of **Indium/m^3 (acceptor** atom**)**:

b) Compute the **density of electrons and holes** on such semiconductor.

c) The mobilities of electrons and holes on Germanium at ***300 K*** are, respectively, μ_n**=0,390 (m^2/Vs)** and μ_p**=0,182 (m^2/Vs).** Compute the **conductivity** of semiconductor in case b), and that of intrinsic semiconductor.

d) **Reason** if the net electric charge of semiconductor in the before cases is positive, negative, or neutral.

12. **(2,5 puntos)** Un semiconductor de Germanio, a 300 K, está **dopado** con **2·10^22 átomos de Arsénico/m^3 (**átomo **donador)** (n_i=2,36·10^19 m^-3 en el Germanio a 300K)

a) Calcula la **concentración de electrones y huecos** en dicho semiconductor.

Si el **dopado es de 2·10^20 átomos de Indio/m^3 (átomo aceptor)**:

b) Calcula la **concentración de electrones y huecos** en dicho semiconductor.

c) Las movilidades de electrones y huecos en el Germanio a ***300 K*** son respectivamente, μ_n **= 0,390 (m^2/Vs)** y μ_p **= 0,182 (m^2/Vs). C**alcula la **conductividad** del semiconductor en el caso b), y la del semiconductor intrínseco.

d) **Razona** si la carga eléctrica neta del semiconductor en los casos anteriores es positiva, negativa, o neutra.

Solution:

a) To find n and p, we have to solve the system of equations given by the mass action law and the electric netrality law:
$$\left.\begin{array}{r} n\cdot p=n_i^2\\ N_A+n=N_D+p\end{array}\right\}$$

But in this case, as the doping is very high compared with the intrinsic density (10^{22}>>>10^{19}) and N_A=0

$n\approx N_D=2\cdot 10^{22}\ e^-/cm^3$ and $n\cdot p=n_i^2=2{,}36^2\cdot 10^{38}\Rightarrow p=\frac{n_i^2}{n}=\frac{2{,}36^2\cdot 10^{38}}{2\cdot 10^{22}}=2{,}78\cdot 10^{16}\ h/cm^3$

b) Now, the density of donor impurities is of the same order than intrinsic density ($10^{19}\approx 10^{20}$), N_A =2·10^20 cm^-3 and N_D =0

Therefore, by solving the system of equations given by the mass action law and the electric netrality law:

$$\left.\begin{array}{r} n\cdot p=(2{,}36\cdot 10^{19})^2\\ p=2\cdot 10^{20}+n\end{array}\right\}\Rightarrow\left\{\begin{array}{l} n=0{,}27\cdot 10^{19}e^-/cm^3\\ p=20{,}6\cdot 10^{19}\ h/cm^3\end{array}\right.$$

c) The conductivity of a semiconductor comes from $\sigma=q_e\left(n\mu_n+p\mu_p\right)$:

For case b): $\sigma=q_e\left(n\mu_n+p\mu_p\right)\approx 1{,}6\cdot 10^{-19}\cdot 20{,}6\cdot 10^{19}\cdot 0{,}182=6\ (\Omega m)^{-1}$ (the term corresponding to the electrons has been neglected because it is very low compared with the first one. Anyway, it is also correct its taking in account).

For the intrinsic semiconductor: $\sigma = q_e\left(n\mu_n + p\mu_p\right) \approx 1{,}6\cdot10^{-19}\cdot 2{,}36\cdot10^{19}(0{,}390+0{,}182) = 2{,}16\ (\Omega m)^{-1}$

d) The electric charge of a semiconductor is neutral in any case, because of the electric neutrality law.

Si te examinas sólo de una parte, debes resolver los cuatro problemas de esa parte.
If you are sitting only one part of the exam, then you have to solve the four problems of that part.

Si te examinas de dos partes: *1 and 2: 1,2,3,5,6,7* *1 and 3: 1,2,3,9,11,12* *2 and 3: 5,6,7,9,11,12*
If you are sitting two parts of the exam: 1 and 2: 1,2,3,5,6,7 *1 and 3: 1,2,3,9,11,12* *2 and 3: 5,6,7,9,11,12*

Si te examinas de las tres partes, debes resolver: *1,3,6,7,9,11*
If you are sitting the three parts of the exam, then you have to solve: 1,3,6,7,9,11

FORM

Magnetic Forces $\vec{F} = q(\vec{v}\times\vec{B})$ $d\vec{F} = Id\vec{l}\times\vec{B}$ $\vec{\mu} = N\cdot I\cdot\vec{S}$ $\vec{\tau} = \vec{\mu}\times\vec{B}$ $V_H = \dfrac{I\cdot B\cdot d}{n\cdot e\cdot S}$

Sources of magnetic field $d\vec{B} = \dfrac{\mu_0 I}{4\pi}\dfrac{d\vec{\ell}\times\vec{r}}{r^3}$ $\mu_0 = 4\pi 10^{-7}$(I.S.units) $B = \dfrac{\mu_0 I}{2\pi x}$

$B = \dfrac{\mu_0 I}{2R}$ $\oint_L \vec{B}\cdot d\vec{l} = \mu_0 \sum I$ $B = \dfrac{\mu_0 N I}{l}$

Electromagnetic induction $|\varepsilon| = \dfrac{d\phi}{dt}$ $\phi = L\cdot I$ $\phi_{21} = M\cdot I_1$ $W_L = \dfrac{1}{2}L\cdot I^2$

Alternating current $\varphi = \varphi_u - \varphi_i$ $X_L = L\omega$ $X_C = \dfrac{1}{C\omega}$ $U_{rms} = \dfrac{U_m}{\sqrt{2}}$ $I_{rms} = \dfrac{I_m}{\sqrt{2}}$

$tg\varphi = \dfrac{L\omega - 1/C\omega}{R}$ $Z = \dfrac{U_m}{I_m} = \sqrt{R^2 + \left(L\omega - 1/c\omega\right)^2}$

$P(t) = u(t)\cdot i(t) = U_m I_m \cos\varphi \sin^2\omega t + \dfrac{U_m I_m}{2}\sin\varphi \sin 2\omega t$ $P = U_{rms} I_{rms}\cos\varphi$ $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$

Semiconductors $n\cdot p = n_i^2$ $N_A + n = N_D + p$ $\sigma = q_{e^-}(n\mu_n + p\mu_p)$ $q_{e^-} = 1{,}6\cdot10^{-19}\ C$