

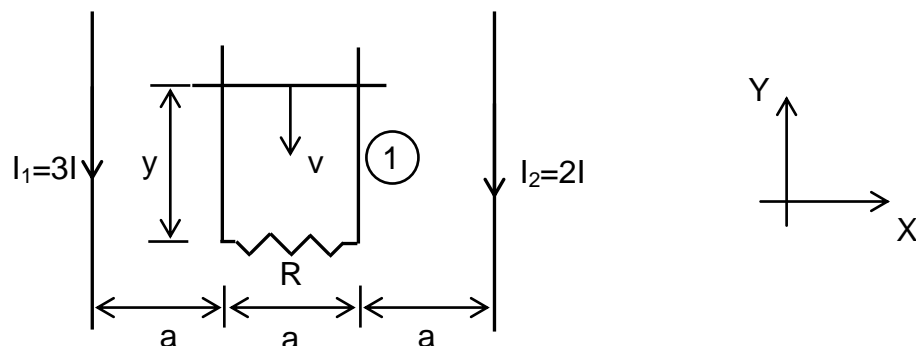


1. (3 points) The upper side of the loop on picture is moving to down with constant speed  $v$ . The loop, whose resistance is  $R$ , is placed in the same plane than two straight, infinite and parallel conductors, as can be seen on picture, flowed by intensities  $I_1=3I$  and  $I_2=2I$  in equal directions. If the time when the height of loop is  $y$  is considered, compute, as a function of  $y$  (0,5 points each):

- Magnetic flux created by the thread 1 through the loop.
- Magnetic flux created by the thread 2 through the loop.
- Induced electromotive force on the loop.
- Induced intensity of current on the loop, giving and justifying its direction.
- Magnetic force acting on side 1 of the loop. Use the reference system on picture.
- Mutual inductance coefficient between thread 1 and loop.

1. (3 puntos) El lado superior de la espira de la figura se mueve hacia abajo con velocidad  $v$  constante. La espira, de resistencia  $R$ , se encuentra en el mismo plano que dos conductores rectilíneos, indefinidos y paralelos, situados como se indica en la figura, por los cuales circulan intensidades  $I_1=3I$  e  $I_2=2I$  en el mismo sentido. Sise considera el instante en que la altura de la espira es  $y$ , calcula, en función de  $y$  (0,5 puntos cada apartado):

- El flujo magnético creado por el hilo 1 a través de la espira.
- El flujo magnético creado por el hilo 2 a través de la espira.
- Fuerza electromotriz inducida en la espira.
- Intensidad de corriente inducida en la espira, indicando y justificando su sentido.
- Fuerza magnética sobre el lado 1 de la espira. Utiliza el sistema de la referencia de la figura.
- Coefficiente de inducción mutua entre el hilo 1 y la espira.



Solution

- The magnetic field created by conductor 1 is out of the page, and then the flux. If  $x$  is the distance from thread 1 to a thin vertical strip on the loop:

$$B_1 = \frac{\mu_0 3I}{2\pi x} \quad \phi_1 = \int_S \vec{B} \cdot d\vec{S} = \int_S \frac{\mu_0 3I}{2\pi x} y dx = \frac{\mu_0 3I y}{2\pi} \int_a^{2a} \frac{dx}{x} = \frac{\mu_0 3I y}{2\pi} \ln 2 \quad \text{Out of the page}$$

- Now, the magnetic field created by thread 2 is into the page. By repeating the calculations:

$$B_2 = \frac{\mu_0 2I}{2\pi x} \quad \phi_2 = \int_S \vec{B} \cdot d\vec{S} = \int_S \frac{\mu_0 2I}{2\pi x} y dx = \frac{\mu_0 2I y}{2\pi} \int_a^{2a} \frac{dx}{x} = \frac{\mu_0 I y}{\pi} \ln 2 \quad \text{Into the page}$$

- $\phi = \phi_1 - \phi_2 = \frac{\mu_0 3I y}{2\pi} \ln 2 - \frac{\mu_0 I y}{\pi} \ln 2 = \frac{\mu_0 I y}{2\pi} \ln 2$  Out of the page

$$|\varepsilon| = \left| \frac{d\phi}{dt} \right| = \frac{d\phi}{dy} \frac{dy}{dt} = \frac{\mu_0 I \ln 2}{2\pi} v$$

d)  $i = \frac{\varepsilon}{R} = \frac{\mu_0 I \ln 2}{2\pi R} v$  The magnetic flux is exiting from the paper. As the surface of the loop is decreasing, this exiting magnetic flux is also decreasing. Therefore, in order to avoid this decreasing of magnetic flux, the induced current must be counterclockwise.

e) The magnetic field at points of side 1 is:  $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 3I}{2\pi 2a} \vec{k} - \frac{\mu_0 2I}{2\pi a} \vec{k} = -\frac{\mu_0 I}{4\pi a} \vec{k}$

The length vector, according the direction of induced current:  $\vec{L} = y\vec{j}$

$$\text{And the force: } \vec{F}_1 = i\vec{L} \times \vec{B} = \frac{\mu_0 I \ln 2}{2\pi R} v \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & y & 0 \\ 0 & 0 & -\frac{\mu_0 I}{4\pi a} \end{vmatrix} = -\left(\frac{\mu_0 I}{2\pi}\right)^2 \frac{v}{2aR} y \ln 2 \vec{i}$$

f) According the definition of M:  $M = \frac{\phi_1}{I_1} = \frac{\mu_0 3I y}{2\pi 3I} \ln 2 = \frac{\mu_0 y}{2\pi} \ln 2$

2. (2 points) Let's consider a coil having **25 cm** length, **1500 turns**, radius **8 cm** and **R=4 Ω**, flowed by an intensity of current **2 A**. A second coil with the same length, **1000 turns**, radius **4 cm** and **R=2 Ω** is coaxially placed inside the first one. By assuming that the magnetic field is uniform inside both solenoids, compute (0,4 points each):

- The **magnetic field** produced by the first coil at a point of its axis.
- The **flux** through the second coil produced by the first one.
- The **mutual inductance coefficient** between both coils.
- The **voltage** on terminals of both solenoids.
- If the current along first coil is varying on time according  $i(t) = 2\cos(500t)$  A, compute the intensity of current flowing along the second solenoid if the terminals of second solenoid are short-circuited.

2. (2 puntos) Sea un solenoide de **25 cm** de longitud, **1500 espiras**, **8 cm** de radio y **R=4 Ω**, por el que circula una corriente de **2 A**. Un segundo solenoide de la misma longitud, **1000 espiras**, **4 cm** de radio y **R=2 Ω** está situado coaxialmente dentro del primero. Asumiendo que el campo magnético es uniforme en el interior de los solenoides, calcular (0,4 puntos cada apartado):

- El **campo magnético** producido por el primer solenoide en un punto de su eje.
- El **flujo** que el primer solenoide produce sobre el segundo.
- El **coeficiente de inducción mutua** entre ambos solenoides.
- La **tensión** en los terminales de ambos solenoides.
- Si la corriente en el primer solenoide varía con el tiempo según la expresión  $i(t) = 2\cos(500t)$  A, calcula la intensidad que circula por el solenoide interior si cortocircuitamos los extremos del mismo.

Solution

a) By considering the magnetic field uniform inside the coil

$$B = \mu_0 \frac{1500}{25 \cdot 10^{-2}} 2 = 120 \mu_0 10^2 = 4,8\pi 10^{-3} T = 15,08 mT$$

b)  $\phi = BNS = 120 \mu_0 10^2 1000 \pi (4 \cdot 10^{-2})^2 = 768 \pi^2 10^{-5} = 75,8 mWb$

c)  $M = \frac{\phi}{I} = \frac{75,8 \cdot 10^{-3}}{2} = 37,9 mH$

d)  $\varepsilon_1 = iR = 2 \cdot 4 = 8V$   $\varepsilon_2 = 0V$

e)  $\phi_2 = Mi_1 = 37,9 \cdot 10^{-3} 2 \cos(500t) = 75,8 \cos(500t) mWb$

$$\varepsilon_2 = \left| \frac{d\phi_2}{dt} \right| = \left| \frac{d}{dt} (75,8 \cdot 10^{-3} \cos(500t)) \right| = 75,8 \cdot 10^{-3} \cdot 500 \sin(500t) = 37,9 \sin(500t) V$$

$$i_2 = \frac{\varepsilon_2}{R} = \frac{37,9 \sin(500t)}{2} = 18,95 \sin(500t) A$$

3. (3 points) The **voltage** applied to a RLC series circuit goes **60° behind the intensity**. The resistor is **20Ω** sized, the voltage on terminals of capacitor is  $u_c(t)=120\cos(1000t-120^\circ)$  V, and the maximum voltage on terminals of capacitor is **twice** of that on inductor. Find (0,6 points each):

- The magnitude of **L** and **C**.
- The instantaneous intensity, **i(t)**.
- The instantaneous voltage on resistor and inductor,  $u_R(t)$  and  $u_L(t)$ .
- The instantaneous voltage **u(t)** on terminals of RLC dipole. Draw the **phasor diagram** of circuit.
- Which **frequency** should have the applied voltage in order the **intensity** flowing along the circuit was the **maximum**?

3. (3 puntos) La **tensión** aplicada a un circuito RLC serie va **retrasada 60° con respecto a la intensidad**. La resistencia es de **20 Ω**, la tensión en los terminales del condensador es  $u_c(t)=120\cos(1000t-120^\circ)$  V, y la tensión máxima en los terminales del condensador es **el doble** de la de la bobina. Calcula (0,6 puntos cada apartado):

- Los valores de **L** y **C**.
- La intensidad instantánea, **i(t)**.
- Las tensiones instantáneas en resistencia y bobina,  $u_R(t)$  and  $u_L(t)$ .
- La tensión instantánea **u(t)** en los terminales del dipolo RLC. Dibuja el diagrama fasorial del circuito.
- ¿Qué **frecuencia** debería tener la tensión aplicada al circuito para que la intensidad que circular por él fuera la máxima posible?

Solution:

a) From the phase lag of dipole  $\varphi = -60^\circ \Rightarrow \operatorname{tg}(-60^\circ) = -\sqrt{3} = \frac{X_L - X_C}{R} = \frac{X_L - X_C}{20} \Rightarrow X_L - X_C = -20\sqrt{3} \Omega$

Moreover  $U_{c\max} = 2U_{L\max} \Rightarrow I_{\max} X_C = 2I_{\max} X_L \Rightarrow X_C = 2X_L$

By solving this system:  $X_L - 2X_L = -20\sqrt{3} \Rightarrow X_L = 20\sqrt{3} \Rightarrow L = \frac{X_L}{\omega} = \frac{20\sqrt{3}}{1000} = \frac{\sqrt{3}}{50} = 34,6 \text{ mH}$

And  $X_C = 2X_L = 40\sqrt{3} \Rightarrow C = \frac{1}{X_C \omega} = \frac{1}{40\sqrt{3} \cdot 1000} = \frac{\sqrt{3}}{12} \cdot 10^{-4} = 14,4 \mu\text{F}$

b)  $\frac{U_{c\max}}{I_{\max}} = X_C \Rightarrow I_{\max} = \frac{U_{c\max}}{X_C} = \frac{120}{40\sqrt{3}} = \sqrt{3} = 1,73 \text{ A}$

$-90^\circ = \varphi_u - \varphi_i \Rightarrow \varphi_i = \varphi_u + 90^\circ = -120^\circ + 90^\circ = -30^\circ$

Therefore:  $i(t) = \sqrt{3} \cos(1000t - 30^\circ) \text{ A}$

c)  $u_R(t) = 20\sqrt{3} \cos(1000t - 30^\circ) = 34,6 \cos(1000t - 30^\circ) \text{ V}$

$u_L(t) = \sqrt{3} \cdot 20\sqrt{3} \cos(1000t + 60^\circ) = 60 \cos(1000t + 60^\circ) \text{ V}$

d)  $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{20^2 + (20\sqrt{3})^2} = 20 \cdot 2 = 40 \Omega$

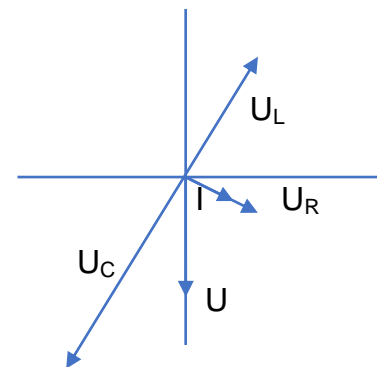
$U_{\max} = Z I_{\max} = 40\sqrt{3} \text{ V}$

$-60^\circ = \varphi_u - \varphi_i \Rightarrow \varphi_u = \varphi_i - 60^\circ = -30^\circ - 60^\circ = -90^\circ$

$u(t) = 40\sqrt{3} \cos(1000t - 90^\circ) = 69,3 \cos(1000t - 90^\circ) \text{ V}$

e) The maximum intensity is reached at the resonant frequency,  $f_0$ :

$$\frac{1}{C\omega_0} = L\omega_0 \Rightarrow \omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{50 \cdot 12 \cdot 10^4}{\sqrt{3} \cdot \sqrt{3}}} = \sqrt{2} \cdot 10^3 \text{ rad/s} \Rightarrow f_0 = \frac{\omega_0}{2\pi} = 225 \text{ Hz}$$



4. (2 points) An extrinsic **n type** semiconductor is made up by Si doped with  $10^{14}$  atoms of **Sb/cm<sup>3</sup>** (Sb is a donor impurity). The intrinsic density of Si at **300 K** is  $n_i=1,5 \cdot 10^{10} \text{ cm}^{-3}$  and at **500 K**  $n_i= 3,7 \cdot 10^{14} \text{ cm}^{-3}$  (0,4 points each).

- Find the **density of electrons and holes** on such semiconductor at **300 K**.
- Find the **density of electrons and holes** on such semiconductor at **500 K**.
- ¿Which would be the **density** of electrons and holes at **300K** if the semiconductor wasn't **doped**?
- If the mobilities of electrons and holes at **300 K** are  $\mu_n = 0,135 \text{ (m}^2/\text{Vs)}$  and  $\mu_p = 0,05 \text{ (m}^2/\text{Vs)}$  respectively, and the charge of electron is  $q_e = 1,6 \cdot 10^{-19} \text{ C}$ , compute the conductivity of semiconductor in case **a**).
- Reason** if the net electric charge of semiconductor, in the three cases, is positive, negative, or neutral.

4. (2 puntos) Un semiconductor extrínseco **tipo n** está formado por Si dopado con  $10^{14}$  **átomos de Sb/cm<sup>3</sup>** (Sb es un donador de e<sup>-</sup>). La concentración intrínseca del Si a **300 K** es  $n_i=1,5 \cdot 10^{10} \text{ cm}^{-3}$  y a **500 K**  $n_i= 3,7 \cdot 10^{14} \text{ cm}^{-3}$  (0,4 puntos cada apartado).

- Calcula la **concentración de electrones y huecos** en dicho semiconductor a **300 K**.
- Calcula la **concentración de electrones y huecos** en dicho semiconductor a **500 K**.
- ¿Cuál sería la **concentración** de huecos y electrones a **300K** si el semiconductor no estuviera **dopado**?
- Si las movilidades de electrones y huecos a **300 K** son respectivamente,  $\mu_n = 0,135 \text{ (m}^2/\text{Vs)}$ ,  $\mu_p = 0,05 \text{ (m}^2/\text{Vs)}$  y la carga del electrón  $q_e = 1,6 \cdot 10^{-19} \text{ C}$ , calcula la conductividad del semiconductor en el caso **a**).
- Razona** si la carga eléctrica neta del semiconductor, en los tres casos, es positiva, negativa, o neutra.

a) To find n and p, we have to solve the system of equations given by the mass action law and the electric neutrality law:

$$\left. \begin{aligned} n \cdot p &= n_i^2 \\ N_A + n &= N_D + p \end{aligned} \right\}$$

But in this case, as the doping is very high compared with the intrinsic density ( $10^{14} \gg \gg 10^{10}$ )

$$n \approx N_D = 10^{14} \text{ e}^- / \text{cm}^3 \text{ and from the mass action law, } n \cdot p = 1,5^2 \cdot 10^{20} \Rightarrow p = \frac{n_i^2}{n} = \frac{1,5^2 \cdot 10^{20}}{10^{14}} = 2,25 \cdot 10^6 \text{ h} / \text{cm}^3$$

b) Now, the density of donor impurities is of the same order than intrinsic density ( $\approx 10^{14}$  in both cases),  $N_A = 0$  and  $N_D = 10^{14} \text{ cm}^{-3}$

Therefore, by solving the system of equations given by the mass action law and the electric neutrality law:

$$\left. \begin{aligned} n \cdot p &= 3,7^2 \cdot 10^{28} \\ n &= 10^{14} + p \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} n &= 4,28 \cdot 10^{14} \text{ e}^- / \text{cm}^3 \\ p &= 3,23 \cdot 10^{14} \text{ h} / \text{cm}^3 \end{aligned} \right.$$

c) If the material is not doped, the densities of electrons and holes are those of the intrinsic density:  $n = p = n_i = 1,5 \cdot 10^{10} \text{ cm}^{-3}$

d) The conductivity of a semiconductor comes from  $\sigma = q_e (n\mu_n + p\mu_p)$ :

$$\sigma = q_e (n\mu_n + p\mu_p) \approx 1,6 \cdot 10^{-19} \cdot 10^{14} \cdot 10^6 \cdot 0,135 = 2,16 \text{ (}\Omega\text{m)}^{-1} \text{ (the term corresponding to the holes has been neglected because it is very low compared with the first one. Anyway, it is also correct its taking in account).}$$

e) The electric charge of a semiconductor is neutral in any case, because of the electric neutrality law.

## FORM

**Magnetic Forces**  $\vec{F} = q(\vec{v} \times \vec{B})$   $d\vec{F} = I d\vec{l} \times \vec{B}$   $\vec{\mu} = N \cdot I \cdot \vec{S}$   $\vec{\tau} = \vec{\mu} \times \vec{B}$   $V_H = \frac{I \cdot B \cdot d}{n \cdot e \cdot S}$

**Sources of magnetic field**  $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$   $\mu_0 = 4\pi 10^{-7} \text{ (I.S.units)}$   $B = \frac{\mu_0 I}{2\pi x}$

$B = \frac{\mu_0 I}{2R}$   $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \sum I$   $B = \frac{\mu_0 NI}{l}$

**Electromagnetic induction**  $|\varepsilon| = \frac{d\phi}{dt}$   $\phi = L \cdot I$   $\phi_{21} = M \cdot I_1$   $W_L = \frac{1}{2} L \cdot I^2$

**Alternating current**  $\varphi = \varphi_u - \varphi_i$   $X_L = L\omega$   $X_C = \frac{1}{C\omega}$   $U_{rms} = \frac{U_m}{\sqrt{2}}$   $I_{rms} = \frac{I_m}{\sqrt{2}}$

$tg\varphi = \frac{L\omega - 1/C\omega}{R}$   $Z = \frac{U_m}{I_m} = \sqrt{R^2 + (L\omega - 1/C\omega)^2}$

$P(t) = u(t) \cdot i(t) = U_m I_m \cos\varphi \sin^2 \omega t + \frac{U_m I_m}{2} \sin\varphi \sin 2\omega t$   $P = U_{rms} I_{rms} \cos\varphi$   $f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$

**Semiconductors**  $n \cdot p = n_i^2$   $N_A + n = N_D + p$   $\sigma = q_e \cdot (n\mu_n + p\mu_p)$   $q_{e^-} = 1,6 \cdot 10^{-19} \text{ C}$