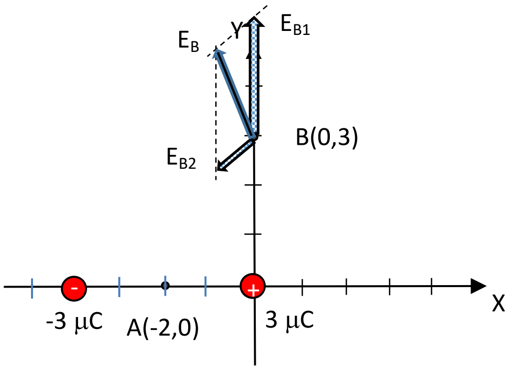
	First mid term FFI exam October, 21th , 2019 Year 2019/20	Applied Physics Dept.

1. (2,5 points) Given the point charges **3 μC** and **-3 μC** on picture, placed at points **(0,0) m** and **(-4,0) m,** compute (0,5 each):



a) The **electric field** vector due to both charges at point B, $\vec{E}_B$. **Drawn** the resulting field.

b) Compute the **electric potential** due to both charges at point A, V_A.

c) The **work** needed to carry a **2 μC** point charge from point A to point B. ¿Is this work done by the forces of the electric field, or against them?

d) If it exists, find a point lying on X axis where the **total electric potential** due to both charges was **zero**, and find the **equipotential surface** with potential **zero**.

e) If it exists, find a point lying on X axis where the **electric field** due to both charges was **zero**.

1. (2,5 puntos) Dadas las cargas puntuales de la figura, de **3 μC** y **-3 μC**, situadas en los puntos **(0,0) m** y **(-4,0) m**, calcula (0,5 puntos cada uno):

a) El vector **campo eléctrico** debido a ambas cargas en el punto B, $\vec{E}_B$. **Dibuja** el campo resultante.

b) El **potencial electrostático** debido a ambas cargas en el punto A, V_A.

c) El **trabajo** necesario para llevar una carga de **2 μC desde el punto A** hasta el punto **B**. Este trabajo ¿es hecho por las fuerzas del campo, o en contra de ellas?

d) Si existe, encuentra un punto sobre el eje X donde el **potencial** electrostático total debido a ambas cargas **sea cero** y halla el **lugar geométrico** de los puntos (superficie equipotencial) de **potencial nulo**.

e) Si existe, encuentra un punto sobre el eje X donde el **campo eléctrico** total debido a ambas cargas **sea cero**.

Solution:

To calculate the electric field or the electric potential, we'll apply the principle of superposition:

a) $$\vec{E}_B = k\frac{3\cdot10^{-6}}{3^2}\vec{j} + k\frac{3\cdot10^{-6}}{5^2}\frac{(-4\vec{i}-3\vec{j})}{5} = 9\cdot10^{9}\cdot3\cdot10^{-6}\cdot(\frac{1}{9}\vec{j}-\frac{4}{125}\vec{i}-\frac{3}{125}\vec{j}) = -864\vec{i}+2352\vec{j}\ N/C$$

b) $$V_A = k(\frac{3\cdot10^{-6}}{2}-\frac{3\cdot10^{-6}}{2}) = 0$$

c) $$V_B = k(\frac{3\cdot10^{-6}}{3}-\frac{3\cdot10^{-6}}{5}) = k(10^{-6}-\frac{3\cdot10^{-6}}{5}) = 9\cdot10^{9}\cdot10^{-6}\cdot\frac{2}{5} = 3600\,V$$

$$W_{AB} = q(V_A - V_B) = 2\cdot10^{-6}(0-3600) = -7,2\cdot10^{-3}\,J$$

As the work is negative, it means that it's done against the forces of the electric field.

d) That point is point A because $V_A = 0$ The equipotential surface V=0 is the plane x=-2.

e) There isn't any point on X axis where the electric field was zero. Only at infinite electric field is null.

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2. (2,5 points) A drop of water (**conductor** material) is spherical with radius **2 mm**, having a net charge **8 nC.**

a) (0,4) Compute the **surface density of charge** of drop.

b) (0,7) By applying Gaus's law, compute the **electric field** at points inside the drop (**r<2 mm**) and outside the drop (**r>2 mm**).

c) (0,7) Compute the **electrostatic potential of drop**.

d) (0,7) The before drop is joined to another equal drop with the same charge, in such way that both of them make up a new spherical drop. Compute the **electrostatic potential of the new drop**.

2. (2,5 puntos) Una gota de agua (material **conductor**) es esférica de radio **2 mm**, y tiene una carga neta de **8 nC.**

a) (0,4) Calcula la **densidad superficial de carga** de la gota.

b) (0,7) Aplicando el teorema de Gauss, calcula el **campo eléctrico** en puntos interiores (**r<2 mm**) y exteriores (**r>2 mm**) a la gota.

c) (0,7) Calcula el **potencial electrostático de la gota**.

d) (0,7) La gota anterior se junta con otra gota idéntica a la anterior y la misma carga, de modo que ambas forman una nueva gota esférica. Calcula el **potencial electrostático de la nueva gota**.

Solution:

a) $\sigma = \frac{q}{S} = \frac{8 \cdot 10^{-9}}{4\pi 4 \cdot 10^{-6}} = \frac{1}{2\pi} 10^{-3} = 0{,}16 \cdot 10^{-3}\ C/m^2$

b) r>2 mm $E 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2} = k\frac{q}{r^2} = 9 \cdot 10^9 \frac{8 \cdot 10^{-9}}{r^2} = \frac{72}{r^2}$

r<2 mm E=0

c) $V = \int_{2\cdot 10^{-3}}^{\infty} \frac{72}{r^2} dr = -\frac{72}{r}\Big|_{2\cdot 10^{-3}}^{\infty} = \frac{72}{2 \cdot 10^{-3}} = 36000\ V$

It can also be computed: $V = k\frac{q}{R} = 9 \cdot 10^9 \frac{8 \cdot 10^{-9}}{2 \cdot 10^{-3}} = 36000\ V$

d) The radius R of new drop will be:

$2 \cdot \frac{4}{3}\pi (2 \cdot 10^{-3})^3 = \frac{4}{3}\pi R'^3 \Rightarrow R' = 2 \cdot 10^{-3} \sqrt[3]{2} = 2{,}52 \cdot 10^{-3}\ m$

And the charge of new drop: Q=8+8=16 nC

Therefore the electric potential of new drop: $V' = k\frac{Q}{R'} = 9 \cdot 10^9 \frac{16 \cdot 10^{-9}}{2{,}52 \cdot 10^{-3}} = 57140\ V$

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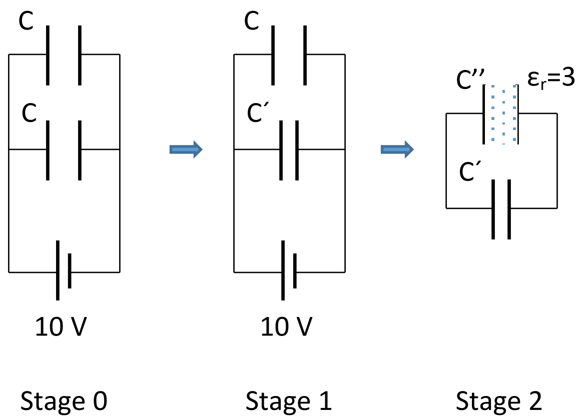
3. (2,5 points) Two **equal capacitors** (capacitance **C**) are connected **in parallel**, and also connected to a power supply giving **10 V** (stage 0).

a) (0,4) Compute the **charge and voltage** on each capacitor (Q_1, Q_2, V_1 and V_2).

With the **power supply connected** to the set, the **plates of second capacitor are approached** up to **a half** of the initial distance (stage 1)

b) (0,7) Compute the **charge and voltage** on each capacitor (Q'_1, Q'_2, V'_1 and V'_2).



Next, the **power supply is disconnected** from the set and a **dielectric** with relative dielectric permittivity **ε_r=3 is inserted** between the plates of **first capacitor** (stage 2)

c) (0,7) Compute the **charge and voltage** on each capacitor (Q''_1, Q''_2, V''_1 and V''_2).

d) (0,7) Compute the **change on the energy stored** by the set of capacitors between the stages 1 and 2. Explain the reason for this change on the stored energy.

3. (2,5 puntos) Dos **condensadores iguales** (capacidad **C**) están conectados **en paralelo**, y también conectados a una fuente de tensión de **10 V** (estado 0)**.**

a) (0,4) Calcula la **carga y el voltaje** en cada condensador (Q_1, Q_2, V_1 y V_2).

Con la **fuente conectada** al conjunto, se **aproximan las placas del condensador 2** hasta una distancia igual a la **mitad de la distancia inicial** (estado 1)

b) (0,7) Calcula la **carga y el voltaje** en cada condensador (Q'_1, Q'_2, V'_1 y V'_2).

A continuación, **la fuente se desconecta** del conjunto, y se inserta un **dieléctrico** de permitividad dieléctrica relativa **ε_r=3 entre las placas del condensador 1** (estado 2)

c) (0,7) Calcula la **carga y el voltaje** en cada condensador (Q''_1, Q''_2, V''_1 y V''_2).

d) (0,7) Calcula el **cambio en la energía almacenada** por el conjunto de condensadores entre los estados 1 y 2. Explica la razón para este cambio en la energía almacenada.

Solution:

a) $Q_1 = Q_2 = C \cdot 10 = 10C \qquad V_1 = V_2 = 10\,V$

b) The new capacitance of capacitor 2 is $C'_2 = \dfrac{\varepsilon_0 S}{\frac{d}{2}} = \dfrac{2\varepsilon_0 S}{d} = 2C$ remaining the voltage constant. Then

$Q'_2 = 2C \cdot 10 = 20C \qquad Q'_1 = C \cdot 10 = 10C \qquad V'_1 = V'_2 = 10\,V$

c) When the power supply is disconnected, the charge of the set remains constant, and the new capacitance of capacitor 1 is $C''_1 = C \cdot \varepsilon_r = 3C$. Therefore, the equivalent capacitance of the set is

$C_{eq} = 2C + 3C = 5C$ and the voltage on both capacitors $V''_1 = V''_2 = \dfrac{Q'_1 + Q'_2}{5C} = \dfrac{30C}{5C} = 6\,V$

And the charges $Q''_1 = 3C \cdot 6 = 18C \qquad Q''_2 = 2C \cdot 6 = 12C$

d) The energy stored on stage 1 is $W' = W'_1 + W'_2 = \dfrac{Q'^2_1}{2C'_1} + \dfrac{Q'^2_2}{2C'_2} = \dfrac{100C^2}{2C} + \dfrac{400C^2}{4C} = 150C$

The energy stored on stage 2 is $W'' = W''_1 + W''_2 = \dfrac{Q''^2_1}{2C''_1} + \dfrac{Q''^2_2}{2C''_2} = \dfrac{18^2 C^2}{6C} + \dfrac{12^2 C^2}{4C} = 90C$

The difference $W'' - W' = -60C$

From stage 1 to stage 2, the set is disconnected from power supply. Therefore, the only reason for this change on the energy is the work done by the system when the dielectric is inserted. The capacitor 1 absorbs the dielectric, and the stored energy decreases 60C.

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4. (2,5 points) Two cylindrical conductors are built from the same material (resistivity ρ) and the same length **L**, and the **ratio of resistances** between both conductors is **R_1/R_2=4**. They are connected **in series**, and a voltage **V=10 V** is applied to the set. Compute (0,5 each):

a) The **ratio of cross sections** between both conductors, **S_1/S_2**.

b) The **ratio of intensities** of current between both conductors, **I_1/I_2**.

c) The **ratio of voltages** between the terminals of both conductors **V_1/V_2**.

d) The **voltage** on terminals of each conductor, **V_1 and V_2**.

e) The **ratio of lost powers** by Joule heating between both conductors, **P_1/P_2**.

4. (2,5 puntos) Dos conductores cilíndricos están hechos del mismo material (resistividad ρ) y la misma longitud **L**, y **la relación entre las resistencias** de ambos conductores es **R_1/R_2=4**. Se conectan **en serie**, y se aplica un voltaje **V=10 V** al conjunto. Calcula (0,5 puntos cada uno):

a) La **relación entre las secciones** de ambos conductores, **S_1/S_2**.

b) La **relación entre las intensidades** de corriente que circulan por ambos conductores, **I_1/I_2**.

c) El **relación entre los voltajes** entre los terminales de ambos conductores, **V_1/V_2**.

d) El **voltaje** en los terminales de cada conductor, **V_1 y V_2**.

e) La **relación entre las potencias disipadas** por efecto Joule en ambos conductores, **P_1/P_2**.

Solution:

a) $R_1 = \frac{\rho L}{S_1} = 4R_2 = 4\frac{\rho L}{S_2} \Rightarrow \frac{S_1}{S_2} = \frac{1}{4}$

b) As both conductors are connected in series, both intensities are equal $\frac{I_1}{I_2} = 1$

c) $\frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{R_1}{R_2} = 4$

d) As both conductors are connected in series $V_1+V_2=10$. From before result, $V_1/V_2=4$ and from these equations:

$$4V_2 + V_2 = 10 \Rightarrow V_2 = 2\,V \qquad \text{and} \qquad V_1 = 8\,V$$

e) $\frac{P_1}{P_2} = \frac{I^2R_1}{I^2R_2} = \frac{R_1}{R_2} = 4$

Form

Electrostatics

$\vec{F} = K\frac{q_1q_2}{r^2}\vec{u}_r$ $\quad \vec{E} = \frac{\vec{F}}{q}$ $\quad K = \frac{1}{4\pi\varepsilon_0} = 9\cdot 10^9 (S.I.)$ $\quad \varepsilon_0 = 8,85\cdot 10^{-12}\ (S.I.)$

$V_A - V_B = \int_A^B \vec{E}\cdot\vec{dr}$ $\quad \vec{E} = K\frac{q}{r^2}\vec{u}_r$ $\quad V = K\frac{q}{r}$ $\quad \int_S \vec{E}\cdot\vec{dS} = \frac{\sum Q}{\varepsilon_0}$ $\quad W_{AB} = q(V_A\text{-}V_B)$

Conductors and capacitors $\quad E = \frac{\sigma}{\varepsilon_0}$ $\quad C = \frac{Q}{V}$ $\quad C = \frac{\varepsilon_0 S}{d}$ $\quad C_{eq} = \sum C_i$

$\frac{1}{C_{eq}} = \sum\frac{1}{C_i}$ $\quad E_d = \frac{E}{\varepsilon_r}$ $\quad C_d = \varepsilon_r C$ $\quad W = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2C}{2}$

Direct Current $\quad \vec{J} = n\cdot e\cdot\vec{v}_d$ $\quad \vec{J} = \sigma\vec{E}$ $\quad R = \frac{V_1 - V_2}{I}$ $\quad R = \rho\frac{L}{S}$

$\rho = \rho_0(1+\alpha(T-T_0))$ $\quad P_R = IV = I^2R = \frac{V^2}{R}$