

1. (2 points) Given the two points charges on picture, 3 nC at point $(0,0) \text{ m}$ and -3 nC at point $(0,4) \text{ m}$,
- Compute the electric field vector at point $A(0,2)$
 - The electric potential at point A .
 - The electric potential at point B .
 - ¿What's the work done by the electric field when moving a point charge of 2 nC from point A to point B ?

1. (2 puntos) Dadas las cargas puntuales de la figura, de 3 nC y -3 nC , situadas en los puntos $(0,0) \text{ m}$ y $(0,4) \text{ m}$:

- Calcula el vector campo eléctrico debido a ambas cargas en el punto A , \vec{E}_A .
- Calcula el potencial electrostático debido a ambas cargas en el punto A , V_A .
- Calcula el potencial electrostático debido a ambas cargas en el punto B , V_B .
- Calcula el trabajo hecho por las fuerzas del campo para llevar una carga de 2 nC desde el punto A hasta el punto B .

Solution:

In order to compute electric field at A , we will apply the superposition principle:

- $$\vec{E}_A = k \left(\frac{3 \cdot 10^{-6}}{2^2} + \frac{3 \cdot 10^{-6}}{2^2} \right) \vec{j} = 9 \cdot 10^9 \frac{6 \cdot 10^{-6}}{4} \vec{j} = \frac{27}{2} 10^3 \vec{j} = 13500 \vec{j} \text{ N/C}$$
- $$V_A = k \left(\frac{3 \cdot 10^{-6}}{2} - \frac{3 \cdot 10^{-6}}{2} \right) = 0$$
- $$V_B = k \left(\frac{3 \cdot 10^{-6}}{3} - \frac{3 \cdot 10^{-6}}{5} \right) = 9 \cdot 10^9 \cdot 3 \cdot 10^{-6} \left(\frac{1}{3} - \frac{1}{5} \right) = 27 \cdot 10^3 \frac{2}{15} = \frac{54}{15} 10^3 = 3600 \text{ V}$$
- $$W_{AB} = q(V_A - V_B) = 2 \cdot 10^{-6} (0 - 3600) = -7,2 \cdot 10^{-3} \text{ J}$$

2. (2 points) Two conductor spheres of radii R and $2R$ are uniformly charged with a charge Q each.

- For sphere 1, calculate the modulus of the electric field at a distance r from the center, inside ($r < R$) and outside ($r > R$). Calculate its potential.
- Both spheres are joined by a conductor wire without capacitance (the wire cannot store charge). Neglecting the electrostatic influence that could exist between them, calculate the charge and potential on each sphere after joining them.
- What is the value of the electric field near the sphere 2 after joining them?

2. (2 puntos) Dos esferas conductoras de radios R y $2R$ están uniformemente cargadas con una carga Q cada una de ellas.

- Para la esfera 1, Calcular el módulo del campo eléctrico a una distancia r del centro, en el interior ($r < R$) y en el exterior ($r > R$). Calcular su potencial.
- Ambas esferas se unen mediante un hilo conductor sin capacidad (el hilo no puede almacenar carga). Despreciando la influencia electrostática que pudiera haber entre ellas, calcula la carga y el potencial en cada esfera después de unir las.
- Cuál es el valor del campo eléctrico en las proximidades de la esfera 2 después de unir las?

Solution:

a) By applying Gauss's law to two spheres with radius r :

$$r < R \quad \phi = \int_{\text{Sphere}} \vec{E} d\vec{S} = E 4\pi r^2 = \frac{0}{\epsilon_0} \Rightarrow E = 0 \qquad r > R \quad \phi = \int_{\text{Sphere}} \vec{E} d\vec{S} = E 4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\text{If we take potential zero at infinite: } V_{\text{esfera}} - V_{\infty} = \int_R^{\infty} \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0 R} \Rightarrow V_{\text{esfera}} = \frac{Q}{4\pi\epsilon_0 R}$$

b) When both spheres are joined, we have a single conductor, being equal the potentials of both spheres. To do that, the charge of one sphere must flow to the other one, but the total charge of the system is preserved. If we call Q_1 and Q_2 the charges of both spheres after joining them and V is their potential:

$$Q_1 + Q_2 = 2Q \quad \text{and} \quad V = \frac{Q_1}{4\pi\epsilon_0 R} = \frac{Q_2}{4\pi\epsilon_0 2R}$$

$$\text{By solving this system: } Q_1 = \frac{2}{3}Q \quad \text{y} \quad Q_2 = \frac{4}{3}Q$$

$$V = \frac{Q_1}{4\pi\epsilon_0 R} = \frac{Q}{6\pi\epsilon_0 R}$$

c) Near the sphere 2, according Coulomb's theorem, the electric field is

$$E = \frac{\sigma_2}{\epsilon_0} = \frac{Q_2}{4\pi(2R)^2 \epsilon_0} = \frac{4Q}{3 \cdot 4\pi 4R^2 \epsilon_0} = \frac{Q}{12\pi\epsilon_0 R^2}$$

3. (2 points) A parallel plate capacitor with capacitance $C = 2 \text{ nF}$ and plates surface $S = 2 \text{ m}^2$ is connected to a voltage source $V_0 = 5 \text{ V}$.

- Compute the value of the electric field between the plates of the capacitor, the surface density of charge on each plate, and the stored energy.
- With the source connected, a dielectric of relative dielectric permittivity $\epsilon_r = 3$ is introduced between the plates. Repeat the same calculations than on before paragraph.
- The capacitor is disconnected from the source, and then the dielectric between its plates is removed. Compute the electric field and the difference of potential between the plates *as well as* the energy stored in the capacitor.
- Compute the distance between the plates of the capacitor.

3. (2 puntos) Un condensador plano se de capacidad $C=2 \text{ nF}$ y superficie de armaduras $S=2 \text{ m}^2$ se conecta a una fuente de tensión $V_0=5 \text{ V}$.

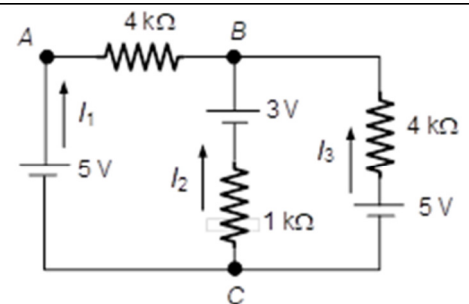
- Calcula el valor del campo eléctrico entre las placas del condensador, la densidad superficial de carga en cada placa, y la energía almacenada.
- Con la fuente conectada, se introduce un dieléctrico de permitividad dieléctrica relativa $\epsilon_r=3$ entre las placas. Repite los mismos cálculos que en el punto anterior.
- El condensador se desconecta de la fuente, y después se retira el dieléctrico entre sus placas. Calcula el campo eléctrico y la diferencia de potencial entre las placas, así como la energía almacenada en el condensador.
- Calcula la separación entre las placas del condensador.

Solution:

- a) $Q = CV_0 = 2 \cdot 5 = 10 \text{ nC}$ $\sigma = \frac{Q}{S} = \frac{10}{2} = 5 \text{ nC/m}^2$ $E = \frac{\sigma}{\epsilon_0} = \frac{5 \cdot 10^{-9}}{8,85 \cdot 10^{-12}} = 564,9 \text{ V/m}$
- $W = \frac{1}{2} CV_0^2 = \frac{2 \cdot 10^{-9} \cdot 25}{2} = 25 \text{ nJ}$
- b) Now, the difference of potential between the plates is preserved: $V' = V_0 = 5 \text{ V}$ and then the electric field remains equal: $E' = E = 564,9 \text{ V/m}$ $C' = 3C = 6 \text{ nF}$ $Q' = C'V_0 = 6 \cdot 5 = 30 \text{ nC}$
- $\sigma' = \frac{Q'}{S} = \frac{30}{2} = 15 \text{ nC/m}^2$ $W' = \frac{1}{2} C'V_0^2 = \frac{6 \cdot 25}{2} = 75 \text{ nJ}$
- c) As the source has been removed, the charge on plates is preserved when the dielectric is removed: $Q'' = Q' = 30 \text{ nC}$
- $C'' = C = 2 \text{ nF}$ $V'' = \frac{Q''}{C} = \frac{30}{2} = 15 \text{ V}$ $E'' = \frac{\sigma''}{\epsilon_0} = \frac{15 \cdot 10^{-9}}{8,85 \cdot 10^{-12}} = 1694,9 \text{ V/m}$
- $W'' = \frac{1}{2} C'V_0^2 = \frac{2 \cdot 15^2}{2} = 225 \text{ nJ}$
- d) As $V = Ed$, it comes that:
- $$15 = 1694,9 \cdot d \Rightarrow d = \frac{15}{1694,9} = 8,85 \text{ mm}$$

4. (2 points) In the circuit on picture:

- Compute the intensities I_1 , I_2 and I_3 with the directions given.
- Compute the difference of potential between points B and C.
- Compute the Thevenin's equivalent generator between points B and C.
- Between points B and C a real generator with electromotive force 2 V and internal resistance $1/3 \text{ k}\Omega$ is connected, so that its positive terminal is connected to point B. Calculate the current that will flow through this new generator.
- Compute the consumed or supplied power by this new generator, and say if it is consumed or generated.



4. (2 puntos) En el circuito de la figura:

- Calcular, con los sentidos mostrados, las intensidades I_1 , I_2 e I_3 .
- Calcular la diferencia de potencial entre los puntos B y C.
- Calcular el generador equivalente de Thevenin entre los puntos B y C.
- Entre los puntos B y C se conecta un generador real de f.e.m. 2 V y resistencia interna $1/3 \text{ k}\Omega$, de manera que su borne positivo queda conectado al punto B. Calcular la intensidad que circulará por este nuevo generador.
- Calcular la potencia consumida o suministrada al circuito por este nuevo generador, y decir si es consumida o suministrada.

Solution:

a. $I_1 + I_2 + I_3 = 0$
 $4I_1 + 3 - 1I_2 - 5 = 0$
 $-4I_2 - 3 - 4I_3 + 5 = 0$

By solving the system: $I_1 = 1/3$ mA $I_2 = -2/3$ mA $I_3 = 1/3$ mA

b. $V_B - V_C = -I_2 + 3 = 2/3 + 3 = 11/3$ V

c.

$$\varepsilon_T = V_B - V_C = \frac{11}{3} \text{ V} \quad \frac{1}{R_T} = \frac{1}{4} + \frac{1}{4} + \frac{1}{1} = \frac{6}{4} \Rightarrow R_T = \frac{2}{3} \text{ k}\Omega$$

d.

$$I = \frac{\frac{11}{3} - 2}{\frac{2}{3} + \frac{1}{3}} = \frac{5}{3} \text{ mA}$$

e. This current will flow through the new generator from B to C, by entering through the positive terminal and exiting through the negative one, consuming energy from the circuit. The consumed power is:

$$P_c = \varepsilon I + I^2 r = 2 \cdot \frac{5}{3} + \frac{25}{9} = \frac{115}{27} \text{ mW}$$

5. (2 points) A current of $I = 5$ A flows through the circuit in the figure. The generator supplies $P_{sg} = 60$ W to the circuit with an efficiency of $\eta_g = 80\%$. Compute:

a) the electromotive force and the internal resistance of *generator*, the power dissipated as heat in the generator and the generated power in the generator.

The motor transforms $P_{tm} = 20$ W into mechanical energy, and has an efficiency of $\eta_m = 80\%$. Compute/Calcula:

b) its contraelectromotive force, its internal resistance, and the power dissipated as heat in the motor.

We substitute the resistance for one of unknown value, the generator by one with electromotive force 30 V and internal resistance 1 Ω , and the motor by one with contraelectromotive force 10 V and internal resistance 2 Ω . When measuring the potential difference between the motor terminals, it turns out to be 20 V. Compute:

c) the current that now travels the circuit, the value of the new resistance and the total power dissipated by Joule heating in the circuit.

5. (2 puntos) Por el circuito de la figura circula una corriente de 5 A. El generador suministra 60 W al circuito con un rendimiento del 80%.

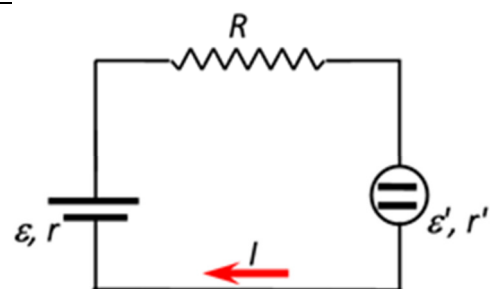
a) Calcula la fuerza electromotriz y resistencia interna del generador, la potencia disipada en forma de calor en el generador y la potencia generada en el generador

El motor transforma 20 W en energía mecánica, y tiene un rendimiento del 80%.

b) Calcular su fuerza contraelectromotriz y su resistencia interna y la potencia disipada en forma de calor en el motor.

Sustituimos la resistencia por otra de valor desconocido, el generador por uno de fuerza electromotriz 30 V y resistencia interna 1 Ω , y el motor por uno de fuerza contraelectromotriz 10 V y resistencia interna 2 Ω . Al medir la diferencia de potencial entre los terminales del motor resulta ser 20 V.

c) Calcula la intensidad que recorre ahora el circuito, el valor de la resistencia y la potencia total disipada en el circuito por efecto Joule.



Solution:

a) If the generator supplies 60 W with 5 A, then:

$$P_s = P_g - P_r = \varepsilon I - I^2 r = 5\varepsilon - 25r = 60$$

Moreover, the efficiency is 80%:

$$\eta = \frac{\varepsilon - Ir}{\varepsilon} = 0,8 \Rightarrow 0,2\varepsilon = 5r$$

By solving the system:

$$5\varepsilon - 25r = 60 \quad \vee \quad \varepsilon = 25r$$

$$5 \cdot 25r - 25r = 100r = 60$$

$$r = 0,6\Omega \quad \varepsilon = 15V$$

b) If the motor transforms 20 W with 5 A, then:

$$P_t = \varepsilon' I = 5\varepsilon' = 20 \Rightarrow \varepsilon' = 4V$$

Moreover, the efficiency is 80%:

$$\eta' = \frac{\varepsilon'}{\varepsilon' + Ir'} = 0,8 \Rightarrow 0,8\varepsilon' + 4r' = \varepsilon' \Rightarrow \varepsilon' = 20r'$$

$$r' = 0,2\Omega$$

c) $V_m = \varepsilon' + Ir' = 10 + I \cdot 2 = 20 \Rightarrow I = 5 A$

$$I = \frac{\varepsilon - \varepsilon'}{R + r + r'} = \frac{30 - 10}{R + 1 + 2} = 5$$

$R = 1\Omega$

$$P_J = I^2 R + I^2 r + I^2 r' = 25(1 + 1 + 2) = 100W$$