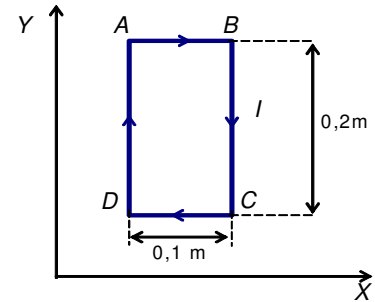




1. Along the loop on picture flows an intensity of current $I = 2 \text{ A}$ in the given direction. The loop is placed inside a magnetic field $\vec{B} = 2\vec{i} + 2\vec{j} \text{ T}$. Find:

- Magnetic forces acting on the four sides of the loop and the total magnetic force acting on the loop
- The magnetic moment of the loop
- Torque of the forces acting on the loop.



1. Sea la espira rectangular de la figura, por la que circula una intensidad $I = 2 \text{ A}$ en el sentido indicado, situada en el interior de un campo magnético $\vec{B} = (2\vec{i} + 2\vec{j}) \text{ T}$. Hallar:

- Fuerzas magnéticas sobre los 4 lados de la espira y Fuerza total sobre la espira
- Momento magnético de la espira,
- Momento resultante de las fuerzas sobre la espira.

Solution:

a) $\vec{F} = I\vec{L} \times \vec{B}$ because B is uniform

$$\vec{F}_{AB} = I\vec{L} \times \vec{B} = 2 * 0,1\vec{i} \times (2\vec{i} + 2\vec{j}) = 0,4\vec{k} \text{ (N)}$$

$$\vec{F}_{CD} = I\vec{L} \times \vec{B} = 2 * 0,1(-\vec{i}) \times (2\vec{i} + 2\vec{j}) = -0,4\vec{k} \text{ (N)}$$

$$\vec{F}_{BC} = I\vec{L} \times \vec{B} = 2 * 0,2(-\vec{j}) \times (2\vec{i} + 2\vec{j}) = 0,8\vec{k} \text{ (N)}$$

$$\vec{F}_{DA} = I\vec{L} \times \vec{B} = 2 * 0,2(\vec{j}) \times (2\vec{i} + 2\vec{j}) = -0,8\vec{k} \text{ (N)}$$

$$\vec{F}_{\text{Total}} = 0 \text{ because it is a closed loop inside a uniform magnetic field}$$

$$\text{b) } \vec{m} = I\vec{S} = 2 * 0,1 * 0,2(-\vec{k}) = -0,04\vec{k} \text{ A.m}^2$$

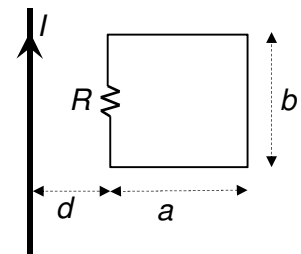
$$\text{c) } \vec{M} = \vec{m} \times \vec{B} = -0,04\vec{k} \times (2\vec{i} + 2\vec{j}) = 0,08\vec{i} - 0,08\vec{j} \text{ (N.m)}$$

2. Along the straight and infinite conductor on picture flows an intensity of current $I = 2\text{sen } \omega t$, where ω is a positive constant and t is the time. In the same plane, as it is shown on picture, there is a loop with resistance R . If we know that $a=d$, compute:

- The magnetic flux through the loop.
- The electromotive force induced on the loop.
- The intensity induced on the loop.
- The mutual inductance coefficient between conductor and loop.

2. Por el conductor rectilíneo de la figura, de longitud infinita, circula una intensidad de corriente $I = 2\text{sen } \omega t$. En el mismo plano y en la posición mostrada, hay una espira de resistencia R . Si $a=d$, calcula:

- El flujo magnético que atraviesa la espira.
- La fuerza electromotriz inducida en la espira.
- La intensidad inducida en la espira.
- El coeficiente de inducción mutua.



Solution:

a) The magnetic field on right of conductor is: $B = \frac{\mu_0 I}{2\pi x}$

and the flux through a surface dS : $d\Phi = BdS = \frac{\mu_0 I}{2\pi x} bdx$

$$\text{The total flux across the loop, } \Phi = \int_d^{d+a} BdS = \int_d^{d+a} \frac{\mu_0 I}{2\pi x} bdx = \frac{\mu_0 I b}{2\pi} \ln \frac{d+a}{d} = \frac{\mu_0 b \text{sen } \omega t}{\pi} \ln 2$$

b) From Faraday's law,
$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt} \frac{\mu_0 b s e n \omega t}{\pi} \ln 2 = \frac{\mu_0 b \omega \ln 2}{\pi} \cos \omega t$$

c)
$$i = \frac{\mathcal{E}}{R} = \frac{\mu_0 b \omega \ln 2}{\pi R} \cos \omega t$$

d)
$$M = \frac{\Phi}{i} = \frac{\mu_0 b}{2\pi} \ln 2$$

3. On a RC series circuit connected to an alternating current generator there is a resistor $R = 10 \Omega$. The voltage on terminals of capacitor is $U_C(t) = 4 \cos(500t - 20^\circ)$ V. The amplitude of the difference of potential on terminals of resistor is 2 V. Find:

- Amplitude of the intensity
- value of C
- The impedance of circuit
- The difference of potential on terminals of generator
- Compute the value L of the inductor that should be connected in series with the circuit, in order the new circuit was a resonant circuit

3. En un circuito RC serie conectado a un generador de corriente alterna, hay una $R = 10 \Omega$, y la tensión instantánea en el condensador de $U_C(t) = 4 \cos(500t - 20^\circ)$ (V). El valor máximo de la diferencia de potencial en bornes de la resistencia es de 2 V. Determina:

- El valor de C.
- La impedancia.
- Intensidad instantánea
- La expresión de la diferencia de potencial en bornes del generador $U(t)$.
- ¿Cuál es el valor de una autoinducción L que se debe colocar en serie para que el circuito esté en resonancia?

Solution:

a) $i_m = \frac{V_R}{R} = 0,2 \text{ A}$

b) $x_C = \frac{U_{Cm}}{i_m} = 20 \Omega \quad C = \frac{1}{x_C \omega} = \frac{1}{10^4} = 100 \mu\text{F}$

c) $Z = \sqrt{R^2 + X_C^2} = \sqrt{500} = 22,4 \Omega$

d) $U_m = Zi_m = 4,47 \text{ V}$ From capacitor $-90^\circ = \varphi_u - \varphi_i = -20 - \varphi_i \rightarrow \varphi_i = 90 - 20 = 70^\circ$

$$\varphi_u - \varphi_i = \text{atan} \frac{-X_C}{R} = \text{atan} \frac{-20}{10} = -63,4^\circ \quad \varphi_u = \varphi_i - 63,4^\circ = 70 - 63,4 = 6,4^\circ$$

$U(t) = 4,47 \cos(500t + 6,4^\circ)$ (V)

e) $x_C = 20 \Omega = x_L = L\omega \rightarrow L = \frac{20}{500} = 0,04 \text{ H} = 40 \text{ mH}$

4. A semiconductor built with Germanium is doped with donor impurities with a concentration of $N_D = \{ND\} \cdot 10^{19} \text{ m}^{-3}$. The intrinsic concentration of such semiconductor at the working temperature is $\{ni\} \cdot 10^{19} \text{ m}^{-3}$ and the mobilities of electrons and holes are respectively $\mu_n = 0.39 \text{ m}^2/\text{Vs}$, $\mu_p = 0.182 \text{ m}^2/\text{Vs}$. The charge of electron is $q_e = 1.6 \cdot 10^{-19} \text{ C}$

- Compute the concentration of electrons and holes on this semiconductor
- Compute the concentration of electrons and holes if the semiconductor wasn't doped

c) Compute the resistivity of semiconductor in case the concentration of electrons in case a)

4. Un semiconductor de germanio está dopado con impurezas donadoras con una concentración de $N_D = 8 \cdot 10^{19} \text{ m}^{-3}$. La concentración intrínseca de portadores a la temperatura de trabajo es $2,36 \cdot 10^{19} \text{ m}^{-3}$ y las movilidades de electrones y huecos a 300 K son respectivamente, $\mu_n = 0,39 \text{ m}^2/\text{Vs}$, $\mu_p = 0,182 \text{ m}^2/\text{Vs}$ y la carga del electrón $q_e = 1,6 \cdot 10^{-19} \text{ C}$,

- Calcula la concentración de electrones y huecos en este semiconductor a 300 K.
- Calcula la concentración de electrones y huecos a 300K si el semiconductor no estuviera dopado.
- Calcula la resistividad del semiconductor en el caso a).

Solution:

a) The donor concentration is of the same order then the intrinsic concentration:

$$\begin{aligned} n &= 8 \cdot 10^{19} + p & pn &= 8 \cdot 10^{19} p + p^2 \\ pn &= 2,36^2 \cdot 10^{38} = 5,57 \cdot 10^{38} & p^2 + 8 \cdot 10^{19} p - 5,57 \cdot 10^{38} &= 0 \end{aligned}$$

$$\begin{aligned} p &= \frac{-8 \cdot 10^{19} \pm \sqrt{64 \cdot 10^{38} + 22,3 \cdot 10^{38}}}{2} = 0,64 \cdot 10^{19} \text{ h/cm}^3 \\ n &= \frac{5,57 \cdot 10^{38}}{p} = 8,64 \cdot 10^{19} \text{ e/cm}^3 \end{aligned}$$

b) If it is not doped: $n = p = n_i = 2,36 \cdot 10^{19} \text{ m}^{-3}$

c) The conductivity can be computed from: $\sigma = q_e(n\mu_n + p\mu_p) = 5,58 (\Omega\text{m})^{-1}$

and the resistivity: $\rho = \frac{1}{\sigma} = 0,18 \Omega\text{m}$

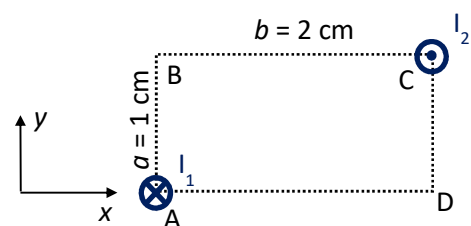
5. The picture shows two straight, parallel and infinite conductors. Along the first one, passing through point A, flows a current $I_1 = 1,8 \text{ A}$. Along the second one, passing through point C, flows a current $I_2 = 2,2 \text{ A}$. In both cases, the directions of current are given.

- Compute the magnetic field vector created by the current 1 at point D.
- Compute the magnetic field vector created by the current 2 at point D.
- Compute the magnetic force by unit of length acting on wire passing through point A.
- Compute the magnetic force by unit of length acting on wire passing through point C.

$$\mu_0 = 4\pi 10^{-7} \text{ TmA}^{-1}$$

5. La figura representa dos hilos rectilíneos, paralelos e indefinidos. Por el primero circula una corriente $I_1 = 1,8 \text{ A}$, y pasa por el punto A. Por el segundo circula una corriente $I_2 = 2,2 \text{ A}$ y pasa por el punto C.

- Calcula el vector campo magnético producido por la corriente 1 en D.
- Calcula el vector campo magnético producido por la corriente 2 en D.



- Calcula la fuerza magnética por unidad de longitud que actúa sobre el hilo que pasa por A.
- Calcula la fuerza magnética por unidad de longitud que actúa sobre el hilo que pasa por C.

Solution:

$$\text{a) } \vec{B}_{1D} = \frac{\mu_0 I_1}{2\pi b} \vec{u}_\varphi \times \vec{u}_r = -\frac{4\pi \cdot 10^{-7} I_1}{2\pi \cdot 0,02} \vec{j} = -18 \vec{j} \mu\text{T}$$

$$\text{b) } \vec{B}_{2D} = \frac{\mu_0 I_2}{2\pi a} \vec{u}_\ell \times \vec{u}_r = \frac{4\pi \cdot 10^{-7} I_2}{2\pi \cdot 0,01} \vec{i} = 44\vec{i} \text{ } \mu T$$

$$\text{c) } \vec{B}_A = \frac{\mu_0 I_2}{2\pi d} \vec{u}_\ell \times \vec{u}_r = \frac{\mu_0 I_2}{2\pi d \sqrt{5}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -1 \\ -2 & -1 & 0 \end{vmatrix} = \frac{2 \cdot 10^{-7} I_2}{\sqrt{5} \cdot 10^{-2}} \frac{\vec{i} - 2\vec{j}}{\sqrt{5}} = 8,8(\vec{i} - 2\vec{j}) \text{ } \mu T$$

$$\vec{F}_A = I_1 \vec{L} \times \vec{B}_A = I_1 L (-\vec{k}) \times 8,8(\vec{i} - 2\vec{j}) = 8,8 I_1 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -1 \\ 1 & -2 & 0 \end{vmatrix} = -15,84(2\vec{i} + \vec{j}) \text{ } \mu N$$

$$\text{d) } \vec{B}_C = \frac{\mu_0 I_1}{2\pi d} \vec{u}_\ell \times \vec{u}_r = \frac{\mu_0 I_1}{2\pi d \sqrt{5}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \frac{2 \cdot 10^{-7} I_1}{\sqrt{5} \cdot 10^{-2}} \frac{\vec{i} - 2\vec{j}}{\sqrt{5}} = 7,2(\vec{i} - 2\vec{j}) \text{ } \mu T$$

$$\vec{F}_C = I_2 \vec{L} \times \vec{B}_C = -I_2 L \vec{k} \times 7,2(\vec{i} - 2\vec{j}) = 7,92 I_2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -1 \\ 1 & -2 & 0 \end{vmatrix} = 15,84(2\vec{i} + \vec{j}) \text{ } \mu N$$