	First mid term FFI exam **November, 30th , 2020** Year 2020/21	Applied Physics Dept.

1. (**2 points**) Given the point charges on picture, 4 nC at point (0,0) m and -1 nC at point (3,0) m:

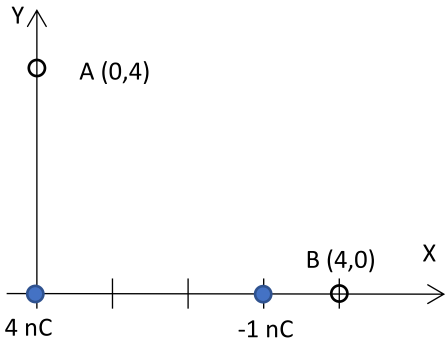
a) Compute the electric field vector at point A(0,4) m.

b) Compute the electric field vector at point B(4,0) m.

c) Find a point where the electric field is zero. Give its coordinates.

d) Compute the work needed to carry a 2 μC point charge from point A to point B. ¿Is this work done by the forces of the electric field, or against them?

e) Find a point lying on X axis (different than point B), where the electric potential is zero.



1. (**2 puntos**) Dadas las cargas puntuales de la figura, 4 nC en el punto (0,0) m y -1 nC en el punto (3,0) m:

a) Calcula el vector campo eléctrico en el punto A (0,4) m.

b) Calcula el vector campo eléctrico en el punto B (4,0) m.

c) Encuentra un punto donde el campo eléctrico se anule. Da sus coordenadas.

d) Calcula el trabajo necesario para llevar una carga de 2 μC desde el punto A hasta el punto B. Este trabajo ¿es hecho por las fuerzas del campo, o en contra de ellas?

e) Encuentra un punto sobre el eje X (distinto del punto B), donde el potencial electrostático se anule.

Solution:

To calculate the electric field or the electric potential, we'll apply the principle of superposition:

a) $$\vec{E}_A = k\frac{4\cdot 10^{-9}}{4^2}\vec{j} + k\frac{1\cdot 10^{-9}}{25}\left(\frac{3\vec{i}-4\vec{j}}{5}\right) = 0,22\vec{i} + 1,96\vec{j} N/C$$

b) $$\vec{E}_B = k(\frac{4}{4^2}-\frac{1}{1^2})10^{-9}\vec{i} = 9\cdot 10^9\cdot(-\frac{3}{4})\cdot 10^{-9}\vec{i} = -\frac{27}{4}\vec{i}\ N/C = -6,75\vec{i}\ N/C$$

c) The electric field can only be null at points on right of charge -1 μC. If x is the X coordinate of such point, must be verified that: $\frac{4}{x^2} = \frac{1}{(x-3)^2}$

This equation has two possible solutions $x_1 = 6\,m \quad and \quad x_2 = 2\,m$, corresponding to points (6,0) m and (2,0) m. Point (2,0) can't be a solution because both electric fields have the same direction at that point. Therefore the solution is point (6,0).

d) $$V_A = k(\frac{4}{4}-\frac{1}{5})\cdot 10^{-9} = 7,2V \qquad V_B = k(\frac{4}{4}-\frac{1}{1})\cdot 10^{-6} = 0$$

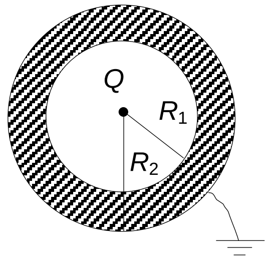
$$W = q(V_A - V_B) = 2\cdot 10^{-6}(7,2-0) = 14,4\cdot 10^{-6}\ J$$

As the work is positive, it means that it is done by the forces of the electric field.

e) The electric potential can be null on right of charge -1 μC (point B) or also between both charges. If we consider a point between both charges with distance x to the origin of coordinates, then: $\frac{4}{x} = \frac{1}{(3-x)} \Rightarrow x = \frac{12}{5} = 2,4\,m$.

So, electric potential is null at point (2,4; 0) m.

2. (2 points) A hollow metallic sphere with inner radius R_1 and outer radius R_2 is linked to ground. A positive point charge Q is placed at the centre of the sphere.



a) What is the charge on inner and outer surfaces of the sphere?

b) Give the electric field as a function of the distance to the centre of the sphere, E(r) for: $r<R_1$ (in the inner space), for $R_1<r<R_2$ (in the metallic sphere), and for $r>R_2$ (outside the sphere).

c) Give the electric potential as a function of the distance to the centre of the sphere, V(r) for: $r<R_1$ (in the inner space), for $R_1<r<R_2$ (in the metallic sphere), and for $r>R_2$ (outside the sphere).

If the ground connection is removed after the charge Q is placed inside the spehre,

d) What is the charge on inner and outer surfaces of the sphere?

2. (2 puntos) La figura muestra una esfera metálica hueca, de radios interior R_1 y exterior R_2, conectada a tierra. Se coloca una carga puntual positiva, *Q*, en el centro de la esfera.

a) ¿Cuál es la distribución de cargas en las superficies interior y exterior de la esfera?

b) Obtén la expresiones de *E*(*r*) para $r< R_1$ (en el hueco interior), $R_1< r< R_2$ (en la esfera), y $r> R_2$ (fuera de la esfera).

c) Obtén la expresiones de *V*(*r*) para $r< R_1$ (en el hueco interior), $R_1< r< R_2$ (en la esfera), y $r> R_2$ (fuera de la esfera).

Si desconectamos la esfera metálica de tierra después de colocar Q en su interior,

d) ¿Cuál es la distribución de cargas en las superficies interior y exterior de la esfera?

Solution:

a) In the inner surface of sphere, because of the electrostatic influence of charge Q, appears a charge –Q, and in the outer surface there is no charge, becuase it is linked to ground.

b) $r<R_1$

By applying Gauss's law to a sphere with radius r:

$$\varphi = \int_{esfera} \vec{E}d\vec{S} = ES = E4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$R_1<r<R_2$ *E=0* because it is a conductor in electrostatic equilibrium.

$r>R_2$ E=0. This system is an electric shield or Faraday's cage. There is no charge creating an electric field outside of the sphere.

c) $R_1<r<R_2$ y $r>R_2$, V=0, because it is a conductor in electrostatic equilibrium linked to ground and without outside charges.

$r<R_1$: By computing the difference of potential between a point with radius r and a point over the inner surface of conductor sphere (radius R_1), along the a field line (straight line L) between r and R_1. And taking in account that potential of sphere is null (V_{R1}=0):

$$V_r - V_{R_1} = V_r = \int_L \vec{E}d\vec{r} = \int_r^{R_1} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R_1}\right)$$

It could also have been solved by applying the superposition principle: the total potential is the summ of potentials due to every charge:

$$V(r) = V_Q(r) + V_{-Q,R_1}(r) + V_{Q',R_2}(r)$$

$$r < R_1 \rightarrow \qquad V(r) = \frac{Q}{4\pi\varepsilon_0 r} + \frac{-Q}{4\pi\varepsilon_0 R_1}$$

$$R_1 < r < R_2 \rightarrow \quad V(r) = \frac{Q}{4\pi\varepsilon_0 r} + \frac{-Q}{4\pi\varepsilon_0 r} = 0$$

$$r > R_2 \rightarrow \qquad V(r) = \frac{Q}{4\pi\varepsilon_0 r} + \frac{-Q}{4\pi\varepsilon_0 r} = 0$$

d) If the sphere is disconnected from ground, the charges remain equal than before.

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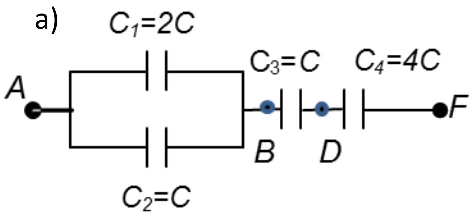
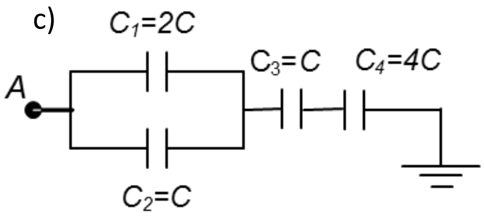
3. (2 points) The set of capacitors on picture is connected to a difference of potential V_{AF}=10V.

a) Compute the charge and voltage on each capacitor (Q_1, Q_2, Q_3, Q_4, V_1, V_2, V_3 and V_4.

b) Compute the equivalent capacitance of the set of capacitors when a dielectric with ε_r=2 is inserted between the plates of capacitor C_1 and the distance between plates of capacitor C_3 is halved.

c) With the conditions of paragraph a), compute the electric potential at points B and D, V_B and V_D if point F is linked to ground and V_A=10V.





3. (2 puntos) La asociación de condensadores de la figura se conecta a una d.d.p. V_{AF}=10V.

a) Calcula la carga de cada condensador y la d.d.p. en bornes de cada condensador.

b) Calcula la capacidad equivalente de la asociación de condensadores si introducimos un dieléctrico de ε_r=2 en el condensador C_1 y reducimos a la mitad la distancia entre las armaduras del condensador C_3.

c) Con las condiciones del apartado a), calcula V_B y V_D si conectamos el punto F a tierra y V_A=10V

Solution:

a) This exercise can be solved in two different ways, oth of them correct:

- Without using the equivalent capacitance:

- Capacitors 1 and 2 are connected in parallel, and then $V_1 = V_2 \Rightarrow \frac{Q_1}{2C} = \frac{Q_2}{C} \Rightarrow Q_1 = 2Q_2$
- Capacitors 3 and 4 are connected in series, and so Q_3=Q_4
- The charge of capacitor 3 is divided into capacitors 1 and 2, and so: $Q_1 + Q_2 = Q_3 \Rightarrow Q_3 = 3Q_2$
- The difference of potential between A and F is 10 V, and so: $V_1 + V_3 + V_4 = \frac{Q_1}{2C} + \frac{Q_3}{C} + \frac{Q_4}{4C} = 10$

By solving: $\frac{2Q_2}{2C} + \frac{3Q_2}{C} + \frac{3Q_2}{4C} = 10 \Rightarrow Q_2 = \frac{40}{19}C \Rightarrow Q_1 = \frac{80}{19}C \Rightarrow Q_3 = Q_4 = \frac{120}{19}C$

$$V_1 = V_2 = \frac{Q_1}{2C} = \frac{40}{19}V \qquad V_3 = \frac{Q_3}{C} = \frac{120}{19}V \qquad V_4 = \frac{Q_4}{4C} = \frac{30}{19}V$$

Obviously is verified that $V_1 + V_3 + V_4 = \frac{40}{19} + \frac{120}{19} + \frac{30}{19} = \frac{190}{19} = 10\,V$

- By using the equivalent capacitance of the set of capacitors. Capacitors 1 and 2 are in parallel, and so $C_{12} = 2C + C = 3C$. C_{12} is in series with C_3 and C_4. Therefore

$$\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{4C} = \frac{19}{12C} \Rightarrow C_{eq} = \frac{12}{19}C$$

The charge of equivalent capacitor is equal to that of C_4 and C_3 and so

$$Q_4 = Q_3 = C_{eq} \cdot 10 = \frac{12}{19} C \cdot 10 = \frac{120}{19} C \qquad V_3 = \frac{Q_3}{C} = \frac{120}{19} V \qquad V_4 = \frac{Q_4}{4C} = \frac{30}{19} V$$

$$V_1 = V_2 = 10 - (V_3 + V_4) = 10 - (\frac{120}{19} + \frac{30}{19}) = \frac{40}{19} V \qquad Q_1 = 2CV_1 = \frac{80}{19} C \qquad Q_2 = CV_2 = \frac{40}{19} C$$

b) With the new conditions C'_1=4C and C'_3=2C. And the new equivalent capacitance is:

$$\frac{1}{C'_{eq}} = \frac{1}{5C} + \frac{1}{2C} + \frac{1}{4C} = \frac{19}{20C} \Rightarrow C'_{eq} = \frac{20}{19} C$$

c) If point F is grounded V_F=0 and V_A=10 V. Therefore

$$V_B = V_A - V_1 = 10 - \frac{40}{19} = \frac{150}{19} V \quad \text{and} \quad V_D = V_B - V_3 = \frac{150}{19} - \frac{120}{19} = \frac{30}{19} V$$

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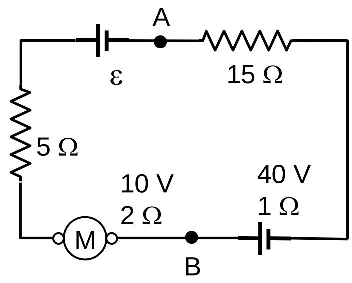
4. (2 points) In the circuit on picture, the device with contraelectromotive force ε is acting as a receptor.



a) Mark the correct equation of such circuit.

If the intensity flowing the circuit is I=1 A, compute:

b) The power consumed on the 15 Ω resistor.

c) The difference of potential between points A and B on circuit.

d) The efficiency of 40 V generator.

e) Total power generated on circuit.

f) Difference of potential between the terminals of motor M.

g) Total power consumed on all the resistors on circuit (even on the internal resistors)

4. (2 puntos) En el circuito de la figura, el elemento con fuerza contraelectromotriz ε actúa como receptor.

a) Indica la ecuación correcta de ese circuito.

Si la intensidad que circula por el circuito es I=1 A, calcula:

b) La potencia consumida por la resistencia de 15 Ω.

c) La diferencia de potencial entre los puntos A y B del circuito.

d) El rendimiento del generador de 40 V.

e) Potencia total generada en el circuito.

f) La diferencia de potencial entre los terminals del motor M.

g) Potencia total consumida en todas las resistencias del circuito (incluyendo las resistencias internas).

Solution:

a) $I = \frac{40 - 10 - \varepsilon}{23}$

b) $P_R = I^2 R = 1^2 \cdot 15 = 15\,W$

c) $V_A - V_B = 1(15+1) - 40 = -24\,V$

d) $\eta_g = \frac{P_s}{P_g} = \frac{\varepsilon I - I^2 r}{\varepsilon I} = \frac{40 \cdot 1 - 1^2 \cdot 1}{40 \cdot 1} = 0{,}97$

e) $P_{Tg} = 40 \cdot 1 = 40\,W$

f) $V_{motor} = \varepsilon' + Ir' = 10 + 1 \cdot 2 = 12\,V$

g) $P_{Tr} = I^2 \cdot \sum r = 1^2 (15 + 1 + 2 + + + 5) = 23\,W$

5. (2 points) Given the circuit on picture

a) Mark the correct equations of such circuit according the Kirchoff's rules, using the directions shown on picture.

b) By assuming that the result for the currents computed from before equations is I1=1 A ,I2=0,75 A and I3=0,25 A, compute the Thevenin's equivalent generator between points B and ground.

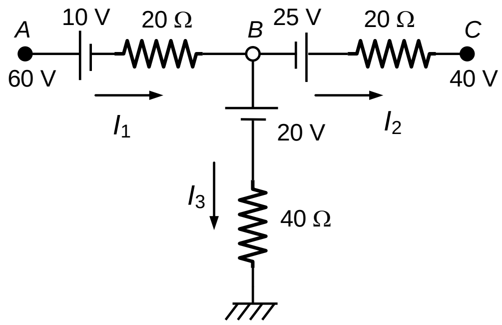
c) If we connect a 4 kΩ resistor between point B and ground in the original circuit, which will be the intensity flowing along this resistor? To find it, you can use the calculated Thevenin's equivalent generator.

5. (2 puntos) Dado el circuito de la figura:

d) Indica qué ecuaciones son las correctas, según las leyes de Kirchoff, de acuerdo con los sentidos de las corrientes de la figura.

e) Asumiendo que el resultado correcto para las corrientes calculadas de las ecuaciones anteriores es I1=1 A ,I2=0,75 A e I3=0,25 A, calcula el Generador equivalente de Thevenin entre B y tierra.

f) Si conectamos una resistencia de 4 kΩ entre el punto B y tierra en el circuito original, ¿qué intensidad circulará por ella?



Solution:

a) $I_1 = I_2 + I_3$ $60=20I_1 + 10 + 20 + 40I_3$ $40=-20I_2 + 25 + 20 + 40I_3$

b) $\varepsilon_T = V_B - V_{tierra} = I_3 40 + 20 = 0{,}25 \cdot 40 + 20 = 30\,V$

$$\frac{1}{R_T} = \frac{1}{20} + \frac{1}{40} + \frac{1}{20} = \frac{5}{40} \Rightarrow R_T = 8\,\Omega$$

c) $I = \dfrac{30}{4 \cdot 10^3 + 8} = 7{,}48\,mA$