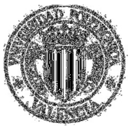
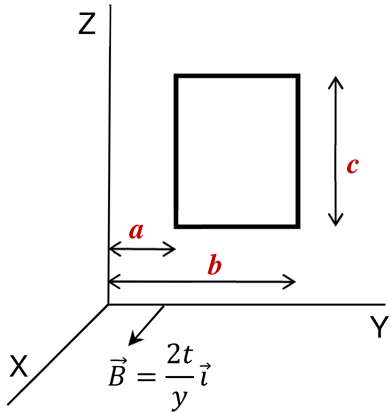
	**Second mid term FFI exam** **February, 1st , 2021** Year 2020/21	Applied Physics Dept.

1. On the loop of picture is acting a magnetic field $\vec{B} = \frac{2t}{y}\vec{\imath}$, being t the time. If ***a=0,1m; b=0,6m; c=0,5m,*** compute on time ***t*=1 s**:
a) The magnetic flux through the surface of the loop.
b) The induced electromotive force on the loop.
c) The induced intensity, giving its direction, if the resistance of the loop is ***R*=100 Ω**
d) The modulus of the magnetic force acting on sides with length *c*, placed at distances *a* and *b* from axis OZ, and the total force acting on the loop.



1. La espira de la figura está sometida a la acción de un campo magnético de expresión $\vec{B} = \frac{2t}{y}\vec{\imath}$, donde t es el tiempo. Si ***a=0,1m; b=0,6m; c=0,5m,*** calcula para el instante ***t*=1s**:
a) El flujo del campo magnético a través de la superficie de la espira.
b) La fuerza electromotriz inducida
c) La intensidad inducida, indicando su sentido, si la resistencia de la espira es ***R*=100 Ω**
d) La fuerza magnética sobre los lados de longitud *c*, situados a distancias *a* y *b* del eje OZ, y la fuerza total sobre la espira.

Solution:
a) We'll take an elemental surface $d\vec{s} = c \cdot dy\vec{\imath}$
The flux crossing this surface (out of paper) is $d\Phi = \vec{B} \cdot d\vec{s} = \frac{2t}{y}\vec{\imath} \cdot c \cdot dy\vec{\imath} = 2t \cdot c\frac{dy}{y}$

By integrating this elementary flux we can calculate the flux across the loop (out of paper):

$$\Phi = 2t \cdot c\int_a^b \frac{dy}{y} = 2t \cdot c \ln\frac{b}{a} = ln6 = 1{,}79\, Wb$$

b) The modulus of the induced electromotive force is:

$$|\varepsilon_i| = \left|\frac{d\Phi}{dt}\right| = \left|2c \cdot ln\frac{b}{a}\right| = \ln 6 = 1{,}79\, V$$

c) And $i = \frac{|\varepsilon_i|}{R} = \frac{ln6}{100} = 0{,}018\, A$

As the magnetic field increases on time, the flux across the loop exiting from the plane of paper will also increase. Therefore, the induced current will try to create a magnetic field opposite to the existing magnetic field. To do that, its direction must be clockwise.

d) Along these sides, the mangnetic field is uniform, and the force acting over them:

$$\vec{F}_1 = i\left(c\vec{k} \times \frac{2t}{0{,}1}\vec{\imath}\right) = 0{,}018 \cdot 0{,}5\frac{2}{0{,}1}\vec{\jmath} = 0{,}18\vec{\jmath}\, N$$

And

$$\vec{F}_2 = i\left(-c\vec{k} \times \frac{2t}{0{,}6}\vec{\imath}\right) = -0{,}018 \cdot 0{,}5\frac{2}{0{,}6}\vec{\jmath} = -0{,}03\vec{\jmath}\, N$$

The magnetic forces on upper and lower sides are equal but opposite, cancelling each other. Therefore, the total force acting on the loop is:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = 0{,}15\vec{\jmath}\, N$$

2. Let's consider a solenoid with length ***L* = 10 cm**, ***N_1* = 1000** turns, and radius ***r* = 0,5 cm** , flowed by an intensity of current ***I* = 1 A** (its resistance can be neglected). A second solenoid with ***N_2*=500** turns, radius ***r_2*=0,4 cm** and ***R*=1 Ω**, is coaxially placed inside the first one in the central area of the first one. By assuming that the magnetic field is uniform inside both solenoids, find:
a) The magnetic field created by the first solenoid at a point of its axis.
b) The flux of the magnetic field created by the first solenoid through the second solenoid.
c) The mutual inductance coefficient between both solenoids.
d) The difference of potential between the terminals of both solenoids.
e) If the current on the first solenoid is varying on time according the equation $i(t) = \frac{t^2}{2}$, compute on time ***t*=1s**, the intensity would flow along the second solenoid if its terminals are short-circuited (joined).

2. Sea un solenoide de ***L* = 10 cm** de longitud, ***N_1* = 1000** espiras, ***r* = 0,5 cm** de radio, por el que circula una corriente de ***I* = 1 A** (su resistencia puede despreciarse). Un segundo solenoide de ***N_2*=500** espiras, ***r_2*=0,4** cm de radio y ***R*=1 Ω**, está situado coaxialmente dentro del primero y en la zona central. Admitiendo que el campo magnético es uniforme en el interior de los solenoides. Calcular:
a) El campo magnético producido por el primer solenoide en un punto de su eje.
b) El flujo del campo magnético a través del solenoide interior.
c) El coeficiente de inducción mutua entre ambos solenoides.
d) La diferencia de potencial en los terminales de ambos solenoides.
e) Si la corriente del primer solenoide varía con el tiempo según la expresión $i(t) = \frac{t^2}{2}$, calcula en el instante ***t*=1s**, la intensidad que circularía por el solenoide interior si unimos sus extremos.

Solution:
a) The magnetic field inside a straight and long solenoid is:

$$B_Z = \frac{\mu_0 N_1}{L} I = \frac{4\pi \cdot 10^{-7} 1000}{0,1} \cdot 1 = 4\pi \cdot 10^{-3}\ T = 12,57\ \text{mT}$$

b) The flux across the second solenoid (by assuming the magnetic field uniform):

$$\Phi = N_2 S_2 B_Z = 500 \cdot \pi \cdot 0,004^2 \cdot 4\pi \cdot 10^{-3} = 0,315\ mWb$$

c) The mutual inductance coefficient: $M = \frac{\Phi}{I} = \frac{0,315}{1} = 0,315\ mH$

d) AS teh intensity on first solenoid is constant, there is no difference of potential between the terminals of any solenoid: $|\varepsilon_{i1}| = |\varepsilon_{i2}| = 0$

e) The intensity on second solenoid will be: $i_2 = \frac{|\varepsilon_{i2}|}{R_2} = \frac{M\frac{di}{dt}}{R_2} = \frac{0,315 \cdot 10^{-3} \cdot t}{1} = 0,315\ mA$

3. A RL series circuit with a self-inductance *L* = 0,05H is connected to an alternating current generator. Along the circuit is flowing a current $i(t) = 5\cos 100t\ (A)$. The maximum difference of potential between the terminals of the resistor is 50 V. Find:

a) The magnitude of *R*.
b) The impedance of such circuit.
c) The maximum voltage on terminals of generator, U_m .
d) Phase lag between voltage on generator and intensity, $\varphi = \varphi_u - \varphi_i$
e) Which capacitor C should be connected in series with the RL circuit in order the new circuit (with the capacitor) was a resonant circuit?

3. En un circuito RL serie conectado a un generador de corriente alterna, con *L* = 0,05H, circula una intensidad de corriente $i(t) = 5\cos 100t\ (A)$. El valor máximo de la diferencia de potencial en bornes de la resistencia es de 50 V. Determina:

a) El valor de *R*.
b) La impedancia del circuito.

c) La tensión máxima en bornes del generador U_m.
d) Desfase entre tensión en el generador e intensidad, $\varphi = \varphi_u - \varphi_i$
e) ¿Cuál es el valor del condensador C que se debe colocar en serie para que el circuito esté en resonancia?

Solution:

a) Thje difference of potential on resistor goes on phase with the intensity, being its amplitude 50 V:

$$v_R(t) = 50\,cos\,100t\,(V)$$

$$R = \frac{V_R}{I} = \frac{50}{5} = 10(\Omega)$$

b) $Z = \sqrt{R^2 + (L\omega)^2} = \sqrt{10^2 + (0{,}05 \cdot 100)^2} = 5\sqrt{5}(\Omega)$
c) $U_m = I_m Z = 5 \cdot 5\sqrt{5} = 25\sqrt{5}\,V$
d) Phase lag is: $\varphi = \tan^{-1}\frac{L\omega}{R} = \tan^{-1}\frac{5}{10} = 26{,}5°$
e) If the circuit must be a resosnant circuit, its reactance must be null: $L\omega - \frac{1}{C\omega} = 0$

$$C = \frac{1}{L\omega^2} = \frac{1}{0{,}05 \cdot 100^2} = 2mF$$

4. A type n extrinsic semiconductor is made up by Si doped with $5{\cdot}10^{14}$ atoms of Sb/cm^3 (Sb is a donor of e^-). The intrinsic concentration of Si at 300 K is n_i=$1{,}5{\cdot}10^{10}$ cm^{-3} and at 500 K, n_i= $3{,}7 \cdot 10^{14}$ cm^{-3}. The mobilities of electrons and holes at 300 K are respectively μ_n = 0,135 (m^2/Vs) y μ_p = 0,05 (m^2/Vs). Find:
a) The concentration of electrons in this semiconductor at 300 K.
b) The concentration of holes in this semiconductor at 500 K.
c) The concentration of electrons and holes at 300 K it the semiconductor wasn't doped.
d) The conductivity of such semiconductor if the concentration of electrons was $8{\cdot}10^{14}$ electrons/cm³ and the temperature 300 K.
e) Say if the net electric charge of semiconductor in the three cases is positive, negative, or neutral.

4. Un semiconductor extrínseco tipo *n* está formado por Si dopado con $5{\cdot}10^{14}$ átomos de Sb/cm^3 (Sb es pentavalente, un dador de e^-). La concentración intrínseca del Si a 300 K es n_i=$1{,}5{\cdot}10^{10}$ cm^{-3} y a 500 K n_i= $3{,}7 \cdot 10^{14}$ cm^{-3}. Las movilidades de electrones y huecos a 300 K son respectivamente, μ_n = 0,135 (m^2/Vs) y μ_p = 0,05 (m^2/Vs). Calcula:
a) La concentración de electrones en este semiconductor a 300 K.
b) La concentración de huecos en este semiconductor a 500 K.
c) La concentración de electrones y huecos a 300K si el semiconductor no estuviera dopado.
d) La conductividad del semiconductor en el caso que la concentración de electrones fuera $8{\cdot}10^{14}$ electrones/cm³ y la temperatura 300 K
e) Di si la carga eléctrica neta del semiconductor en los tres casos es positiva, negativa, o nula.

Solución:
a) $n \cdot p = n_i^2 \Rightarrow n \cdot p = 1{,}5^2 \cdot 10^{20}$
(N_D) is very high compared to the intrinsic concentration (n_i) ($5{\cdot}10^{14}$>>>10^{10}) and therefore

$$n \approx N_D = 5 \cdot 10^{14}\ e^-/cm^3 \qquad p = \frac{n_i^2}{n} = \frac{1{,}5^2 \cdot 10^{20}}{5 \cdot 10^{14}} = 0{,}45 \cdot 10^6\ h/cm^3$$

b) As the concentration of impurities is the equal order than the intrinsic concentration, we'll have to solve the system made up by the mass action law and the law of electrical neutrality (N_A = 0, N_D = $5{\cdot}10^{14}$ cm^{-3}):

$$n \cdot p = n_i^2 \Rightarrow n \cdot p = (3{,}7 \cdot 10^{14})^2 = 1{,}369 \cdot 10^{28}$$

$$N_A + n = N_D + p$$
$$n = 5\cdot 10^{14} + p$$

resulting

$$n = 6{,}96\cdot 10^{14} e^-/cm^3$$
$$p = 1{,}96\cdot 10^{14}\ e^-/cm^3$$

c) If the material is not doped, the concentration of electrons and holes are the intrinsic concentration:

$$n = p = n_i = 1{,}5\cdot 10^{10}\ \text{cm}^{-3}$$

d) The conductivity comes from $\sigma = q_e(n\mu_n + p\mu_p)$

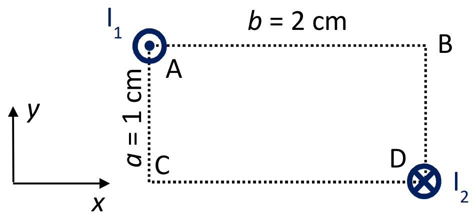
$$\sigma = q_e(n\mu_n) = 1{,}6\cdot 10^{-19}\cdot 8\cdot 10^{20}\cdot 0{,}135 = 17{,}3\ (\Omega\text{m})^{-1}$$

In this case p has been neglected, because it is very low compared to n.

e) The material is neutral in any case (electric neutrality law).

5. The picture shows two straigth, parallel and infinite wires. The first one exits perpendicularly to the plane of paper at point A and it is flowed by a current I_1= 2,5 A. The second one enters perpendicularly to the plane of paper at point D, flowed by a current I_2= 1,6 A. Compute:

a) The magnetic field vector created by current I_1 at point B.
b) The magnetic field vector created by current I_2 at point B.
c) The magnetic force by unit of length acting on wire 1.
d) The magnetic force by unit of length acting on wire 2.



5. La figura representa dos hilos rectilíneos, paralelos e indefinidos. El primero sale perpendicular al papel en el punto A y por él circula una intensidad I_1= 2,5 A, y el segundo entra perpendicularmente al plano del papel en el punto D, y por él circula una intensidad I_2= 1,6 A. Calcula:

a) El vector campo magnético producido por la corriente I_1 en B.
b) El vector campo magnético producido por la corriente I_2 en B.
c) La fuerza magnética por unidad de longitud que actúa sobre el hilo que pasa por A.
d) La fuerza magnética por unidad de longitud que actúa sobre el hilo que pasa por D.

a) $\vec{B}_{1B} = \dfrac{\mu_0 I_1}{2\pi b}\vec{u}_\ell \times \vec{u}_r = \dfrac{4\pi\cdot 10^{-7} I_1}{2\pi\cdot 0{,}02}\vec{j} = 10^2\cdot 10^{-7} I_1\vec{j} = 25\vec{j}\ \mu\text{T}$

b) $\vec{B}_{2B} = \dfrac{\mu_0 I_2}{2\pi a}\vec{u}_\ell \times \vec{u}_r = \dfrac{4\pi\cdot 10^{-7} 1{,}6}{2\pi\cdot 0{,}01}\vec{i} = 20\cdot 10^{-6} 1{,}6\vec{i} = 32\vec{i}\ \mu\text{T}$

c) $\vec{B}_A = \dfrac{\mu_0 I_2}{2\pi d}\vec{u}_\ell \times \vec{u}_r = \dfrac{\mu_0 I_2}{2\pi d}\dfrac{1}{\sqrt{5}}\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & -1\\ -2 & 1 & 0\end{vmatrix} = \dfrac{2\cdot 10^{-7} I_2}{\sqrt{5}\cdot 10^{-2}}\dfrac{\vec{i}+2\vec{j}}{\sqrt{5}} = 4I_2\left(\vec{i}+2\vec{j}\right)\mu\text{T} = 6{,}4\left(\vec{i}+2\vec{j}\right)\mu\text{T}$

$$\vec{F}_A = I_1\vec{L}\times\vec{B}_A = I_1 L\vec{k}\times 4I_2\left(\vec{i}+2\vec{j}\right) = 4I_1I_2\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & 1\\ 1 & 2 & 0\end{vmatrix} = -16\left(2\vec{i}-\vec{j}\right)\ \mu\text{N}$$

d) $\vec{B}_D = \dfrac{\mu_0 I_1}{2\pi d}\vec{u}_\ell \times \vec{u}_r = \dfrac{\mu_0 I_1}{2\pi d}\dfrac{1}{\sqrt{5}}\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & 1\\ 2 & -1 & 0\end{vmatrix} = \dfrac{2\cdot 10^{-7} I_1}{\sqrt{5}\cdot 10^{-2}}\dfrac{\vec{i}+2\vec{j}}{\sqrt{5}} = 4I_1\left(\vec{i}+2\vec{j}\right)\mu\text{T} = 10\left(\vec{i}+2\vec{j}\right)\mu\text{T}$

$$\vec{F}_D = I_2\vec{L}\times\vec{B}_D = -I_2L\vec{k}\times 4I_1\left(\vec{i}+2\vec{j}\right) = 4I_1I_2\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & -1\\ 1 & 2 & 0\end{vmatrix} = 16\left(2\vec{i}-\vec{j}\right)\ \mu\text{N}$$